Introduction to Water Waves and Beach Erosion

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Locations of photos on the front cover.

Negril	Morant Point	Morant Bay	Port Royal
Annotto Bay	Town Centre in Annotto Bay	Hellshire Beach	Hope Bay
Hope Bay near the Swift River	Plumb Point Lighthouse	west beach of Rocky Point	east beach of Rocky Point
Pagee Beach and Cabarita Island in Mort Maria	St. Magarets Bay	Long Bay in Portland Parish	Buff Bay
Alligator Pond	Alligator Pond River mouth	beach near Norman Manley International Airport	Old Harbour

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List of Symbols

a : wave amplitude ($=H/2$)
c : wave celerity or wave phase velocity ($=L/T$)
c_g : group velocity ($=nc$)
E : wave energy density ($=\rho g H^2/8$)
f : frequency ($=1/T$)
H : wave height ($=2a$)
g : gravitational acceleration
h : water depth
K_s : shoaling coefficient
k : wave number ($=2\pi/L$)
L : wavelength ($=cT$)
n : ratio of wave group velocity to wave celerity ($=c_g/c$)
p : pressure
T : wave period
u : horizontal component of velocity in x direction
v : horizontal component of velocity in y direction
w : vertical component of velocity in z direction
x : coordinate in the horizontal direction
y : coordinate in the horizontal direction
z : coordinate in the vertical direction
η : free surface displacement
σ : angular frequency ($=2\pi/T$)
ρ : density of fluid

suffix

$_0$: indicates the value in deepwater, for example H_0 and L_0.
$_i$: indicates the value of the incident wave, for example η_i.
$_{\max}$: indicates the maximum value, for example $H_{\max}$.
$_{\min}$: indicates the minimum value, for example $f_{\min}$.
$_r$: indicates the value of the reflected wave, for example η_r.

others

$\overline{overline}$: indicates the averaged value in time for example, $\overline{E}$ or in numbers for example, $\overline{H}$.

1 Introduction

Jamaica is an island country which is 248 km long and 84 km wide and has an area of 10,911 km^2. The highest elevation is 2,256 m at the peak of Blue Mountains. Jamaica has coastline of 1,022 km long. Although this is quite short compared with that of the Japanese archipelago, which has coastline of about 34,000 km long, Jamaica's coastline is diversified with beach, cliff, rocky shore, mangrove and swamp as shown in **Fig.** 1. Although the result of the coral reef is not exactly the same as that shown in **Fig.** 2, the nearshore zone is divided into two types. One is surrounded by fringing reef as shown with light blue area in **Fig.** 1. The rest of coastline is open sea without fringing reef. The open sea expands from White House to Rocky Point along the mid southern coast and also along the areas in front of Yallahs, Buff Bay and near Port Maria. There are rivers discharging into these areas. In these areas, corals do not grow because of the low temperature of river water compared with that of the seawater.

In order to preserve beaches in Jamaica focusing on both the environment and development, coastal engineering is an indispensable subject for island countries. This textbook aims at introducing the basic knowledge of the water waves from the point of view of the coastal engineering. The stance of describing this document is between a textbook and a glossary of technical terms. Therefore, equations are simply described without any derivation. In addition to water waves, the basic knowledge of beach erosion is outlined and beaches in Jamaica are also contained with some example of beach erosion and beach formation as well as collections of photos.

Figure 1: Jamaican beach typologies (Government of Jamaica, 2017) [1].

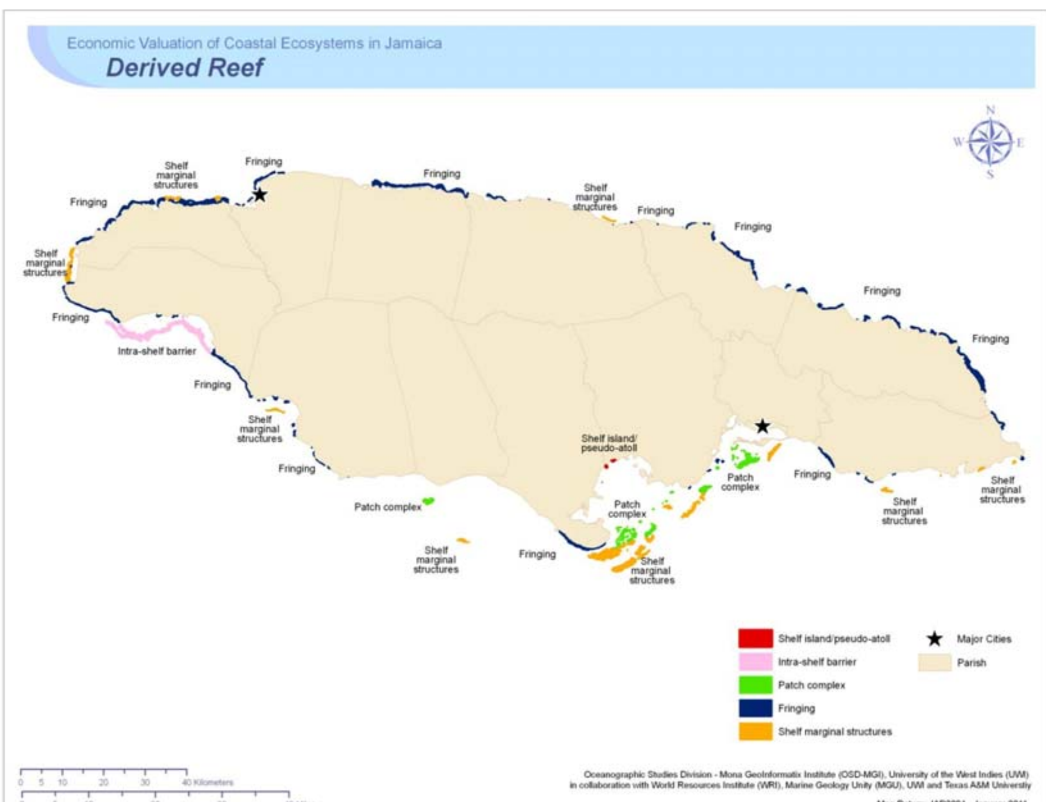


Figure 2: Reef dataset (Maxam *et al.*, 2011)[2].

2 Water Waves

Waves in the ocean are random waves or irregular waves. However, in order to understand motions of water waves the kinematics of monochromatic wave or regular wave based on the **linear wave theory**(also called **Airy wave theory**) is described in this section. The linear wave theory is one of the wave theories, which was derived with an assumption of a small wave amplitude. When the wave amplitude is large compared with the water depth, the nonlinear wave theory is required to describe accurate wave motions. The nonlinear wave theories are only demonstrated to compare two wave theories in this section.

2.1 Regular Waves

2.1.1 Definitions of water waves

Figure 3 shows the definitions of water wave with symbols used in textbooks of coastal engineering. The **wave height** denoted by H is defined as the vertical distance between the **wave crest** and the **wave tough**. The **wavelength** denoted by L is the horizontal distance between the same phase, for example, from the wave crest to the wave crest as shown in **Fig.** 3. The **water depth** is denoted by h and sometimes denoted by d simply coming from depth. When we see waves at a fixed point, sea surface continues going up and down and coming towards us. The time from a certain wave crest to the next wave crest is called the **wave period** denoted by T. The speed of the propagation of wave profiles is called the **wave celerity** denoted by c, which is given as $c = L/T$. The ratio of the wave height to the wavelength, H/L is called the **wave steepness**.

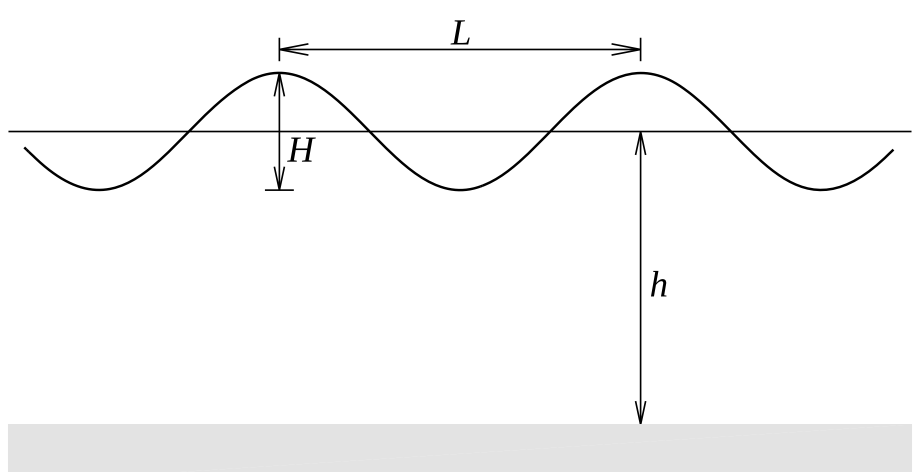


Figure 3: Definitions of water wave.

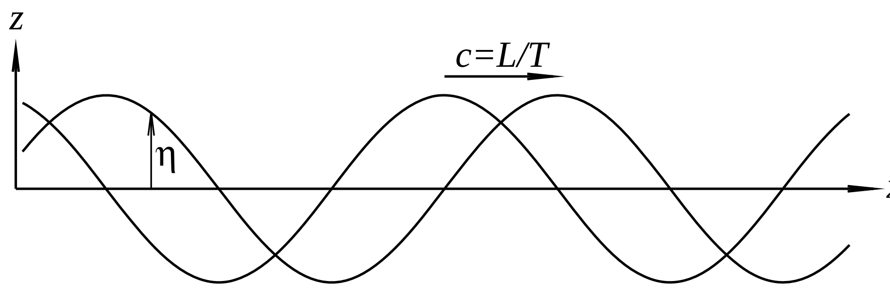


Figure 4: Wave propagation.

The simplest expression of waves is a sinusoidal wave expressed by a sine function or a cosine function. These periodic functions have a period of 2π. In the case of the water wave, the free surface displacement denoted by η is expressed by the following equation based on the linear wave theory.

$$\eta = \frac{H}{2}\cos(\frac{2\pi}{L}x - \frac{2\pi}{T}t) = a\cos(kx - \sigma t) \tag{1}$$

where η is the water level measured from the fixed level such as the mean water level. The equation in the middle is expressed using the wave height H, the wavelength L and the wave period T. It shows that the η is a periodic function both of the wavelength L in space and of the wave period T in time. The **wave amplitude** a is defined by a half of the wave height, $a = H/2$. The term in the parentheses $(kx - \sigma t)$ is called the **phase**, where $k = 2\pi/L$ is the **wave number** and $\sigma = 2\pi/T$ the **angular frequency**. The negative sign of σt means that the wave propagates in the positive direction of the the horizontal coordinate x.

The linear wave theory gives the following relationship between the wave number k and the angular frequency σ with the water depth h, which is called the **dispersion relation**.

$$\sigma^2 = gk\tanh(kh) \tag{2}$$

Equation (2) is also expressed as follows

$$\left(\frac{2\pi}{T}\right)^2 = g\left(\frac{2\pi}{L}\right)\tanh\left(\frac{2\pi h}{L}\right) \tag{3}$$

When the water depth h and the wave period T are given, the wavelength L is obtained by solving the dispersion relation, **Eq. (2)** or **Eq. (3)**.

Equation (2) is an implicit function of k. It is required to solve it by using an iteration method. **Equation** (2) is approximated in the form of the explicit function of the following equation **Eq. (4)**(Beji, 2013)[3]. **Figure** 5 shows an exact solution of the dispersion relation

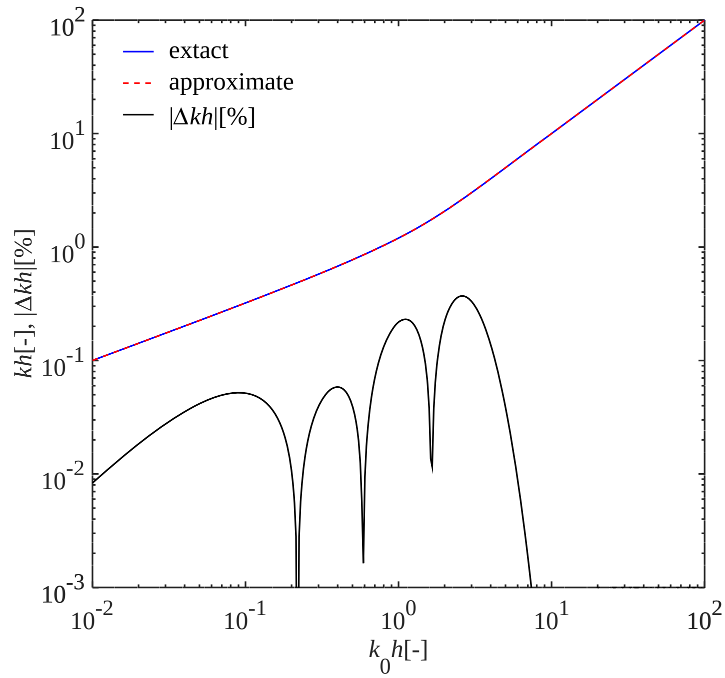


Figure 5: Dispersion relationship

given by **Eq. (2)** with a blue curve. The approximated solution by **Eq. (4)** is shown with a red dashed curve. The error is also shown by $|\Delta kh|$ in % in **Fig.** 5 which is less than 0.5%.

$$kh = \frac{k_0 h[1 + (k_0 h)^{1.3} \exp[-(1.1 + 2.0 k_0 h)]}{\sqrt{\tanh(k_0 h)}} \tag{4}$$

where $k_0 = 2\pi/L_0$: the wave number in the deep water. Although **Eq. (4)** is useful for practical purpose, the author recommends that engineers should use the dispersion relation to keep the physical relationship between k and σ in mind.

From the dispersion relation given by **Eq. (2)**, the wavelength L is expressed as follows:

$$L = \frac{gT^2}{2\pi} \tanh \frac{2\pi h}{L} \tag{5}$$

The wavelength in the deep water is denoted by L_0 and given by the following equation **Eq. (6)**.

$$L_0 = \frac{gT^2}{2\pi} \tag{6}$$

which is obtained by the fact that $\tanh(2\pi h/L) \to 1$ as h/L tends to infinity.

Table 1: Wave classification depending on relative water depth

relative depth h/L	wave type	wavelength	wave celerity
$h/L < 1/20$	long wave	$L = T\sqrt{gh}$	$c = \sqrt{gh}$
$1/20 < h/L < 1/2$	shallow wave	$L = \frac{gT^2}{2\pi}\tanh\frac{2\pi h}{L}$	$c = \frac{gT}{2\pi}\tanh\frac{2\pi h}{L}$
$1/2 < h/L$	deep water wave	$L_0 = \frac{gT^2}{2\pi}$	$c_0 = \frac{gT}{2\pi}$

2.1.2 Deep water wave, shallow water wave and long wave

The ratio of the water depth to the wavelength noted by h/L is called the **relative water depth**. The $kh = 2\pi h/L$ is equivalent to the relative water depth and shown in the dispersion relation of **Eq. (2)**. Considering the limiting values of kh, we can classify waves as follows: $kh \to \infty$ the deep water wave or surface wave, $kh \to 0$ the long wave. The wave between the deep water wave and the long wave is the shallow water wave. **Table 1** shows equations of the wavelength and the wave celerity depending on the relative water depth h/L and the wave types.

$$
\begin{array}{rl}
1/2 < h/L & \text{the deep water wave} \\
1/20 < h/L < 1/2 & \text{the shallow water wave} \\
h/L < 1/20 & \text{the long wave}
\end{array}
$$

2.1.3 Velocity of wave

Figure 6 shows an example of profiles of horizontal component of wave velocity. It is given by the following equation, **Eq. (7)**. At the bottom of the sea, the horizontal component of velocity is not null because the free slip condition is imposed on the wave theory. The vertical component of wave velocity is given by **Eq. (8)**. At the bottom of the sea, the vertical component of that is null because the fluid does not penetrate into the seabed.

$$u = a\sigma\frac{\cosh k(h+z)}{\sinh kh}\cos(kx - \sigma t) \tag{7}$$

$$w = a\sigma\frac{\sinh k(h+z)}{\sinh kh}\sin(kx - \sigma t) \tag{8}$$

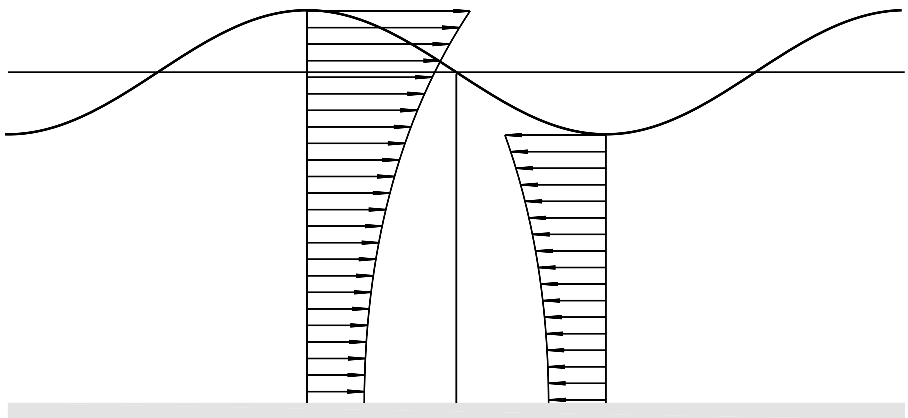


Figure 6: Velocity profile of horizontal component.

2.1.4 Motion of water particle

As shown in **Fig.** 4 the waves propagate with the wave celerity c. However, it is noted that the water itself does not flow with the wave celerity in the direction of wave propagation. Under the water surface, the water itself makes an orbital path. It moves back and forth during one wave period. Here we introduce a concept of the water particle. It does not mean the molecular size of water. In order to grasp the fluid motion of waves, we introduce the conceptual size of the water to depict the streamline or the motion of the fluid. We call it the water particle.

Figure 7 shows water particle motion with blue circles as well as the free surface. The water particles at the different levels move according to the wave motion. At the the time of $t = 0*T/40$, they line up on the vertical position under the wave crest. As the time passes from the left top to the bottom and to the right top to the bottom in **Fig.** 7, the water particles move from the highest position to the lower and right positions up to the the time of $t = 10*T/40$. After passing the zero up crossing level, the water particles move to the left, which is the opposite direction of the wave propagation. At the time of $t = 20 * T/40$, the free surface displacement is at the level of the wave trough and the water particles make a vertical line again. Moving to the higher and left position, they return to the original positions at the time of $t = 40 * T/40$ (the same as the figure at $t = 0 * T/40$).

Figure 8 shows paths of the water particle motions. **Figure** 8 (a), (b) and (c) show typical patterns of the water movement of the deep water wave, of the shallow water wave and of the long wave, respectively. In the case of the deep water wave, the wave motion does not reach the sea bottom. The deepwater wave is not affected by the seabed. Therefore, the deepwater wave is also called the surface wave. In cases of the shallow water waves and the long waves as shown in

Fig. 8 (b) and (c), the path of water particle motion at the sea bottom is no longer an orbit. It moves just back and forth without a vertical shift.

The water particle motion is expressed with the following two equations to give the horizontal and vertical positions, respectively.

$$x - x_0 = -a\frac{\cosh k(h+z_0)}{\sinh kh}\sin(kx_0 - \sigma t) \tag{9}$$

$$z - z_0 = a\frac{\sinh k(h+z_0)}{\sinh kh}\cos(kx_0 - \sigma t) \tag{10}$$

From **Eq. (9)** and **Eq. (10)**, the following equation is obtained:

$$\frac{(x-x_0)^2}{\left\{a\dfrac{\cosh k(h+z_0)}{\sinh kh}\right\}^2} + \frac{(z-z_0)^2}{\left\{a\dfrac{\sinh k(h+z_0)}{\sinh kh}\right\}^2} = 1 \tag{11}$$

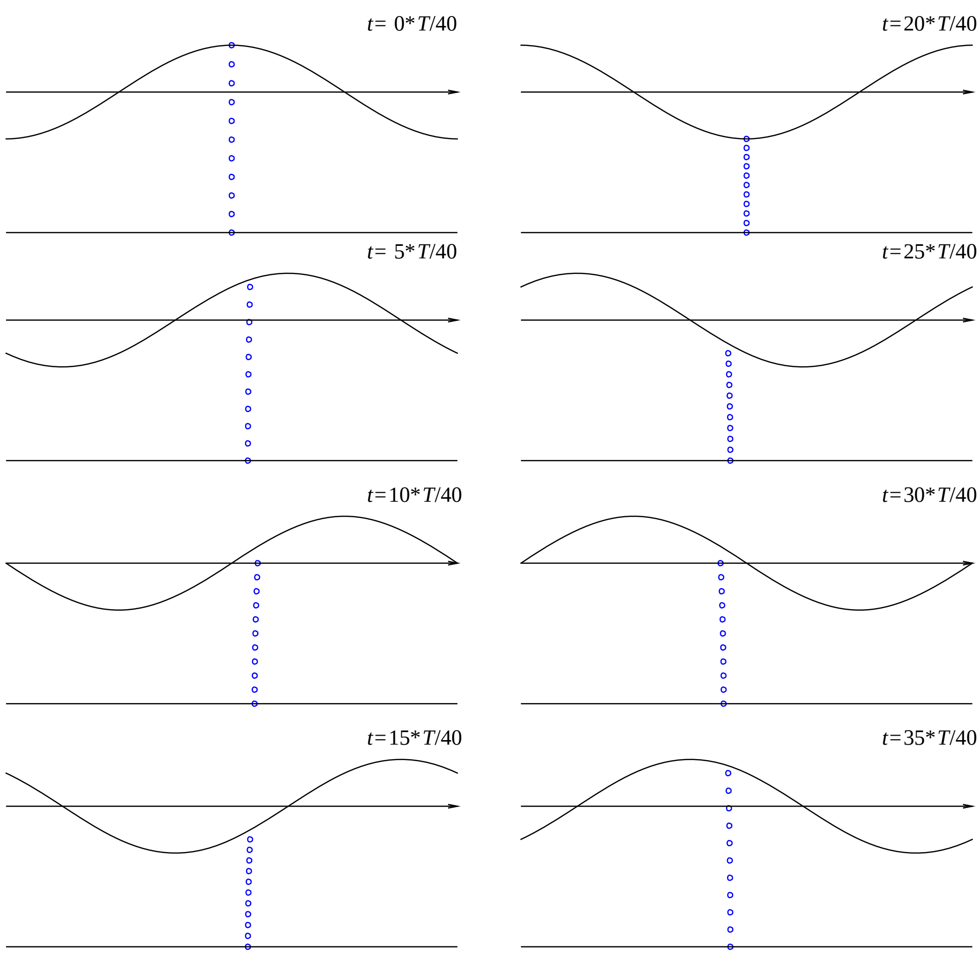
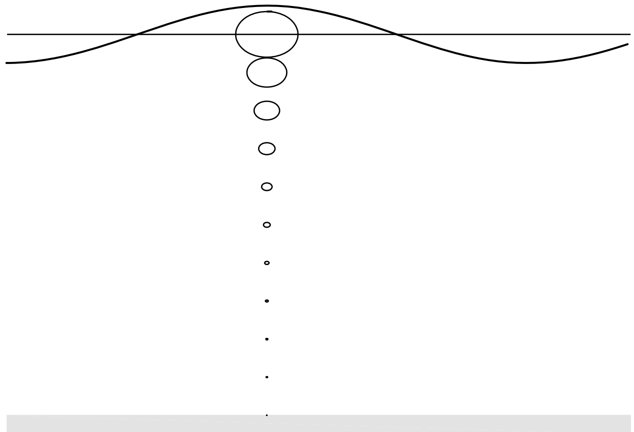
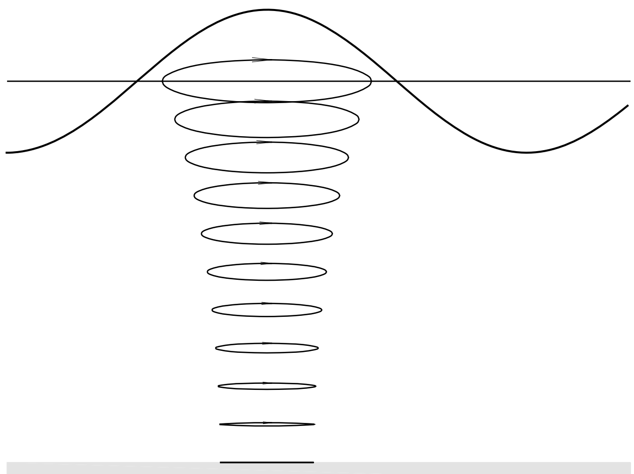


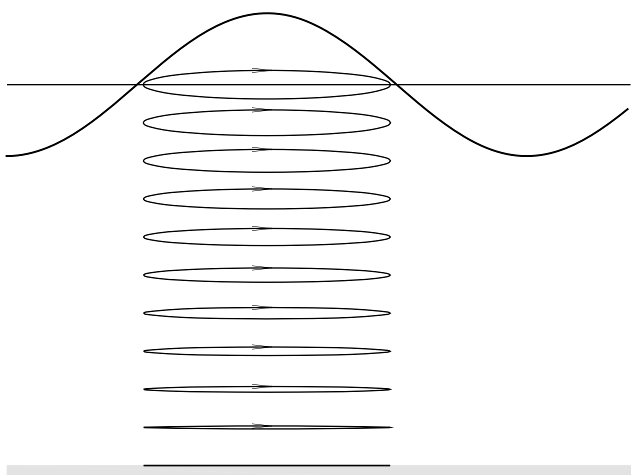
Figure 7: Orbital motion of water particles due to wave.



(a) Deep water wave



(b) Shallow water wave



(c) Long wave

Figure 8: Water particle motion due to waves (figures are not scaled).

2.1.5 Nonlinear waves

When the wave amplitude is small, the linear wave theory is applied. When the wave amplitude becomes large, the nonlinear wave theory gives accurate solutions of the wave motion. There are some nonlinear wave theories. The free surface displacement η of the nonlinear wave theory based on the Stokes wave theory is expressed by the following equation.

$$\eta(x,t) = \sum_{n=1}^{\infty} a_n \cos n(kx - \sigma t) \tag{12}$$

When n is equal to one, the wave theory is equivalent to the linear wave theory. In other words, the nonlinear wave theory contains the shorter waves (nk) and the higher frequency ($n\sigma$) waves. Note that **Eq. (12)** indicates each component of $a_n \cos n(kd - \sigma t)$ propagates with the same wave celerity of $c = \sigma/k$. It is noted that the dispersion relation of the nonlinear wave theory is is different from that of the linear wave theory.

There are some nonlinear wave theories. **Figure** 9 shows a comparison of the free surface displacement between the Stokes wave and the linear wave theory. **Figure** 10 shows a comparison of the free surface displacement between the cnoidal wave and the linear wave theory (The cnoidal wave is a nonlinear and exact periodic wave solution of the Korteweg-de Vries equation). The difference of the free surface displacement between the nonlinear wave and the linear wave is that wave crest becomes higher and the wave trough is smaller of the nonlinear wave.

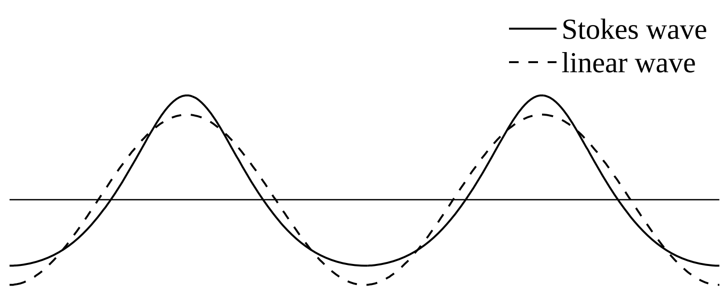


Figure 9: Comparison of profile between Stokes wave and linear wave.

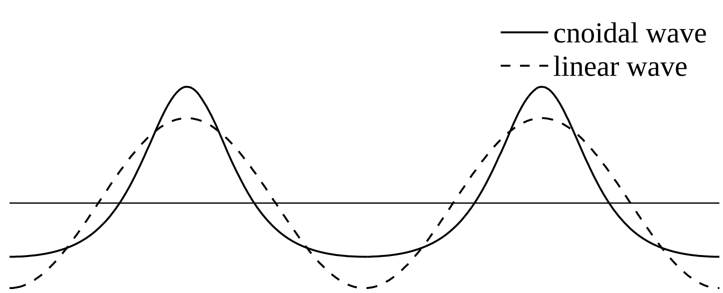


Figure 10: Comparison of profile between cnoidal wave and linear wave.

2.2 Pressure of Wave

2.2.1 Hydrostatic pressure

As an introduction to the water pressure, the hydrostatic pressure is explained first. **Figure** 11 shows a tank filled with fluid such as fresh water or sea water. The difference is the density ρ. The density of fresh water is 1000 kg/m^3 and that of sea water about 1030 kg/m^3 at the normal temperature. When the fluid in the tank is stationary, we call the pressure in that condition the **hydrostatic pressure** here denoted by p_s. The hydrostatic pressure p_s is proportional to the distance from the surface and given by the following equation:

$$p_s = -\rho g z \tag{13}$$

where ρ is the fluid density, g is the gravitational acceleration and z is the vertical coordinate with the positive directing upward. The pressure is a scalar, whereas the velocity and acceleration are vectors which have a magnitude and a direction.

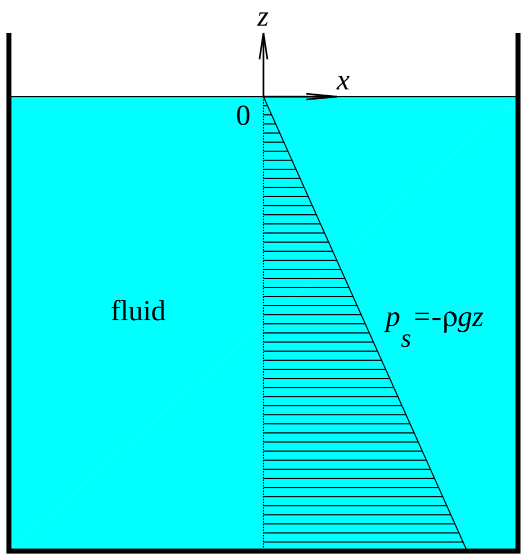
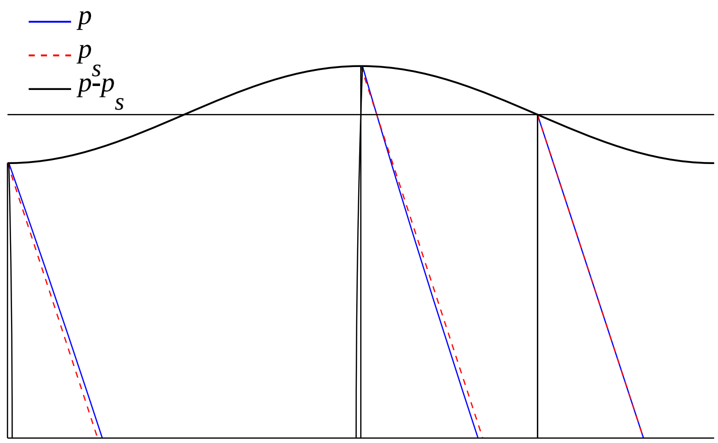


Figure 11: Hydrostatic pressure profile

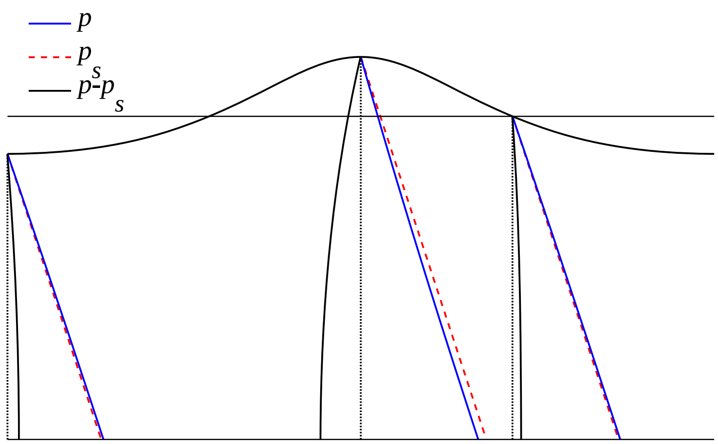
2.2.2 Dynamic pressure

The pressure in waves fluctuates as a function of time and space. The linear wave theory gives the following equation for the pressure.

$$p = \rho g \frac{\cosh k(h+z)}{\cosh kh} \eta - \rho g z \tag{14}$$



(a) Linear wave.

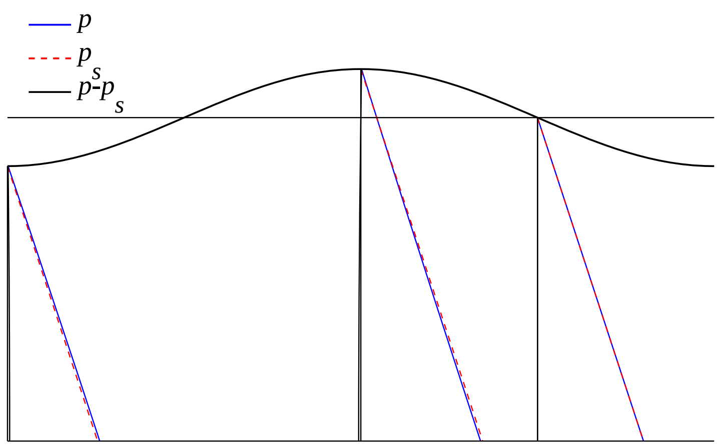


(b) Stokes wave.

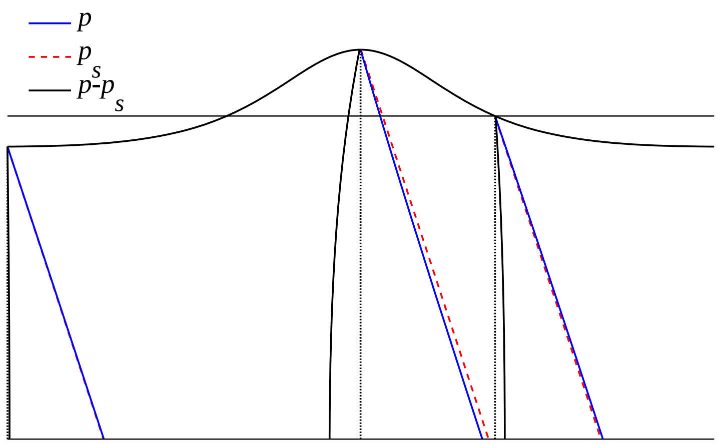
Figure 12: Pressure profiles.

The first term of the right hand side is the dynamic pressure due to the wave motion and η is given by **Eq. (1)**. The second term is the hydrostatic pressure.

Figure 12 shows the comparison of the profiles of pressure under the different locations such as under the wave trough, the wave crest and the still water level. **Figure** 12(a) shows those given by the linear wave theory. The pressure under the wave trough is higher than the hydrostatic pressure while that under the wave crest is lower than the hydrostatic pressure. Under the still water level, the wave pressure is equal to the the hydrostatic pressure. **Figure** 12(b) shows those by the Stokes wave theory. The difference of the pressures between two under the wave trough is smaller than that given by the linear wave theory. The difference under the wave crest is bigger than that given by the linear wave theory. Under the still water level, the wave pressure is not equal to the the hydrostatic pressure. Those are caused by the curvature of the free surface, which



(a) Linear wave.



(b) cnoidal wave.

Figure 13: Pressure profiles.

was resulted in by the centrifugal force.

Figure 13 shows the similar comparisons between the linear wave theory and the cnoidal wave theory. The difference between the Stokes theory and the cnoidal wave theory seems to be small. However, the nonlinear wave theory should be applied under appropriate wave conditions.

2.3 Wave Shoaling

As waves approaching shore, the water depth decreases. Due to the shallowing, waves transform their profiles and the wave celerity changes. The relationship between the water depth and the wave celerity is also expressed with the dispersion relation given by **Eq. (2)**. The wavelength decreases as the water depth becomes shallow. Since the wave period remain constant, the wave celerity also decreases. As for the wave height, from the deep water to the shallow water, it

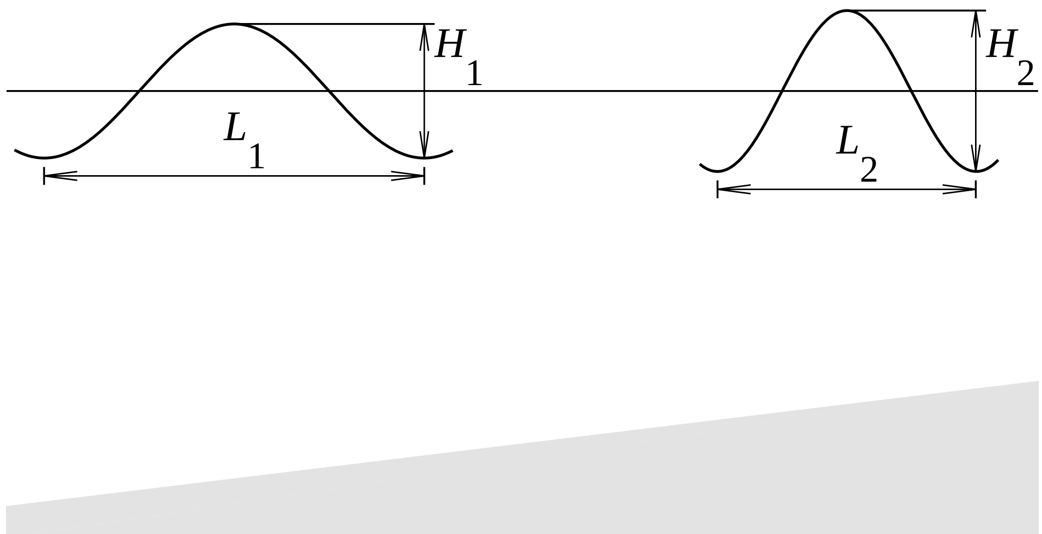


Figure 14: Wave shoaling

decreases at first to about 90% and afterward it increases until waves break. It is important to estimate the wave height at a given water depth when we evaluate beach erosion or design shore protection works. These phenomena are illustrated in **Fig.** 14. The wave dependency on the water depth is called the **wave shoaling**.

We should be noted that the wave celerity is the speed of the propagation of the wave profile and it is not the velocity of water particle of fluid itself. In addition to the wave celerity, we have to learn the group velocity of waves. The **group velocity** is the velocity of propagation of wave energy. The group velocity is usually denoted by c_g and given by the following equation:

$$c_g = \frac{1}{2}\left(1 + \frac{2k}{\sinh(2k)}\right) c = nc \tag{15}$$

The wave energy flux F_x which is the wave energy transported per unit width and per unit time is given by the following equation.

$$F_x = \overline{E} c_g = \overline{E} cn \tag{16}$$

where E is the wave energy per unit time and unit area and is given by **Eq. (17)**.

$$\overline{E} = \frac{1}{8}\rho g H^2 \tag{17}$$

The conservation law of the wave energy flux leads to the wave height change depending on the water depth. As the wave energy flux at two different locations is preserved, the following relation is given:

$$\frac{1}{8}\rho g H^2 cn = \frac{1}{8}\rho g H_0^2 c_0 n_0 \tag{18}$$

where the energy flux on the right hand is set at the deepwater. Since $n_0 = 1/2$ which is the ratio of the group velocity to the wave celerity at the deepwater, the wave height H at a certain water depth is obtained by the following equation:

$$\frac{H}{H_0} = \sqrt{\frac{1}{2n}\frac{c_0}{c}} = K_s \tag{19}$$

$$n = \frac{1}{2}\left[1 + \frac{4\pi h/L}{\sinh(4\pi h/L)}\right] \tag{20}$$

where K_s is called the **shoaling coefficient**. H_0 is the wave height at the deep water which should be given together with the wave period when we examine beach erosion or design shore protection works.

The shoaling coefficient K_s given by **Eq. (19)** with **Eq. (20)** is derived based on the linear wave theory. When the nonlinearity of the wave is significant, the shoaling coefficient is larger than that of the linear wave theory dependent on the wave steepness at the deepwater H_0/L_0. The shoaling coefficient based on the nonlinear long wave theory is given by the following equations:

$$\begin{aligned} &\frac{gHT^2}{h^2} \le 30 && \text{linear wave theory} \\ 30 \le &\frac{gHT^2}{h^2} \le 50 && Hh^{2/7} = \text{const.} \\ 50 \le &\frac{gHT^2}{h^2} && Hh^{5/2}\left[\sqrt{\frac{gHT^2}{h^2}} - 2\sqrt{3}\right] = \text{const.} \end{aligned} \tag{21}$$

Figure 15 shows the shoaling coefficient, K_s as a function of the ratio of the water depth to the deepwater wavelength h/L_0. The relationship of the local relative water depth h/L and the ratio of the water depth h to the deepwater wavelength L_0 denoted by h/L_0 is given by **Eq. (22)**.

$$\frac{h}{L_0} = \frac{h}{L}\tanh\frac{2\pi h}{L} \tag{22}$$

As the wave steepness H_0/L_0 increases, the shoaling coefficient starts increasing in the deeper water region. It means the wave nonlinearity appears. In **Fig.** 15, the curve with the wave steepness H_0/L_0=0.0 is equivalent to the shoaling coefficient of the linear wave theory.

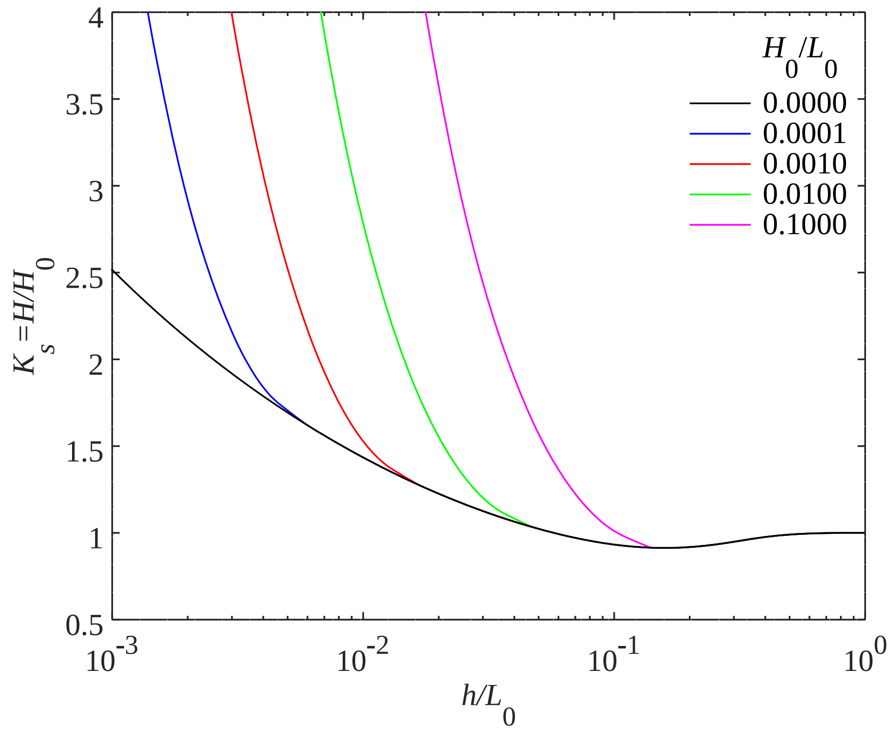


Figure 15: Shoaling coefficient

2.4 Wave Breaking

As the water depth decreases, the wave height H increases in shallow water and the wavelength L decreases due to the wave shoaling remaining the wave period T constant. In other words, the wave steepness H/L increases as the water depth decrease. However, the wave height does not increase infinitely of course. Waves break at a certain condition as we can see them breaking from beach. On a coral reef beach, we can see waves breaking at the fringe of the coral reef because the water depth becomes shallow in short distance there. That tells the most dominant factor on the wave breaking is the ratio of the wave height to the water depth, H/h. One of old wave breaking criteria for the wave breaking condition is expressed by the ratio of the wave height and the water depth as given by **Eq. (23)**.

$$\frac{H}{h} \approx 0.8 \tag{23}$$

Although **Eq. (23)** shows the rule of thumb for the wave breaking condition, the reality of the wave breaking conditions is not that simple, unfortunately.

One of the definitions on the wave breaking is shown in **Fig.** 16 and given by the following equation.

$$u_c > c \tag{24}$$

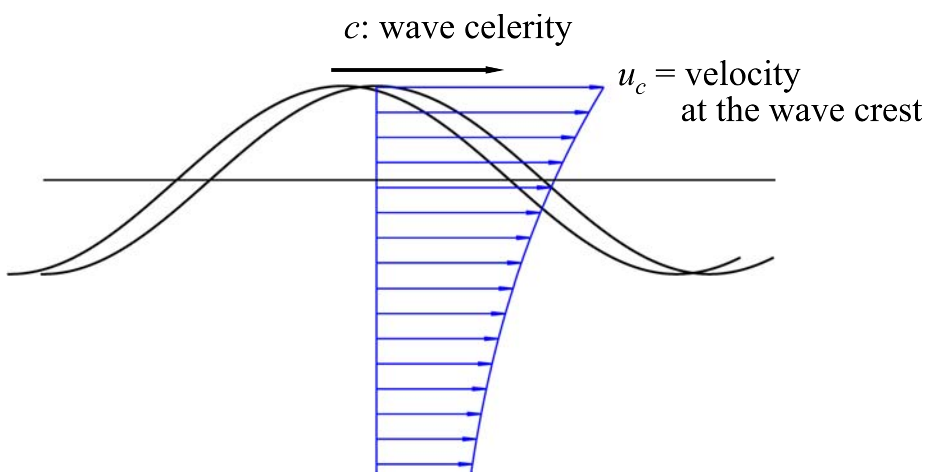


Figure 16: Definition of wave breaking with velocity at wave crest and wave celerity

where u_c is the velocity at the wave crest. When the the velocity at the wave crest u_c exceeds the wave celerity c, the water particle does not remain on the free surface. It leaps out of the free surface. That is the kinematic condition for the wave breaking. Readers should refer to textbooks on the coastal engineering when they need more details on the wave breaking conditions.

On the beach, we can see different types of breaking waves, we call them wave breaker, mainly dependent of the beach slope and wave condition. The breakers are classified mainly into three types, spilling breaker, plunging breaker and surging breaker as shown in **Fig.** 17. These are classified with the surf similarity parameter, denoted by ξ_0 and ξ_b as shown in **Table 2**.

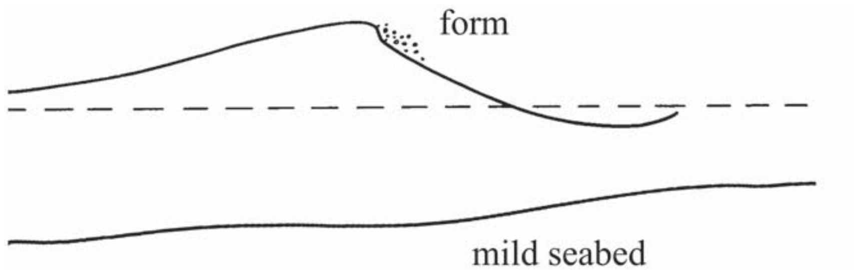
$$\xi_0 = \frac{\tan\beta}{\sqrt{H_0/L_0}} \tag{25}$$

$$\xi_b = \frac{\tan\beta}{\sqrt{H_b/L_0}} \tag{26}$$

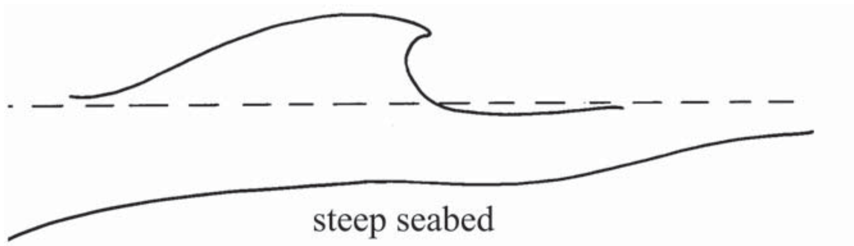
Table 2: Wave breaking criteria

spilling breaker	plunging breaker	collapsing
$\xi_0 > 3.3$	$3.3 > \xi_0 > 0.46$	$0.46 > \xi_0$
$\xi_b > 2.0$	$2.0 > \xi_b > 0.4$	$0.4 > \xi_b$

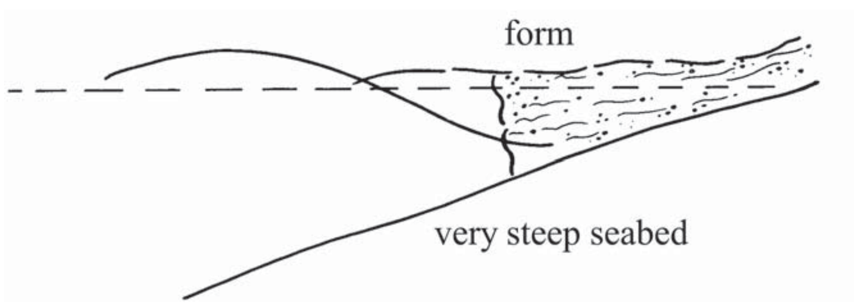
These breaker types affect how sediment is transported and consequently what amount of sediment is. For example, plunging breakers pick up a huge amount of sediment into water column compared with spilling breakers.



(a) Spilling breaker



(b) Plunging breaker



(c) Surging breaker

Figure 17: Breakers

2.5 Standing Waves

In the previous subsections we consider the wave which has one component and propagates in one direction. When there is a sea wall, for example Kingston Water Front in Kingston Harbour, waves are reflected. Those make a combination of incident waves and reflected waves and it is called the **standing waves**.

An incident wave denoted by η_i and a reflected wave η_r have different propagation directions. These two waves are expressed by the following equations, respectively.

$$\eta_i = a\cos(kx - \sigma t) \tag{27}$$

$$\eta_r = a\cos(kx + \sigma t) \tag{28}$$

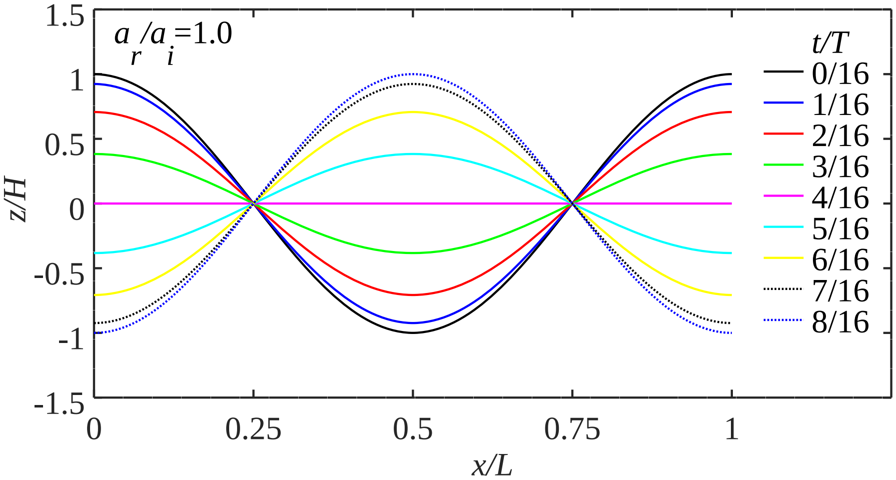


Figure 18: Standing wave

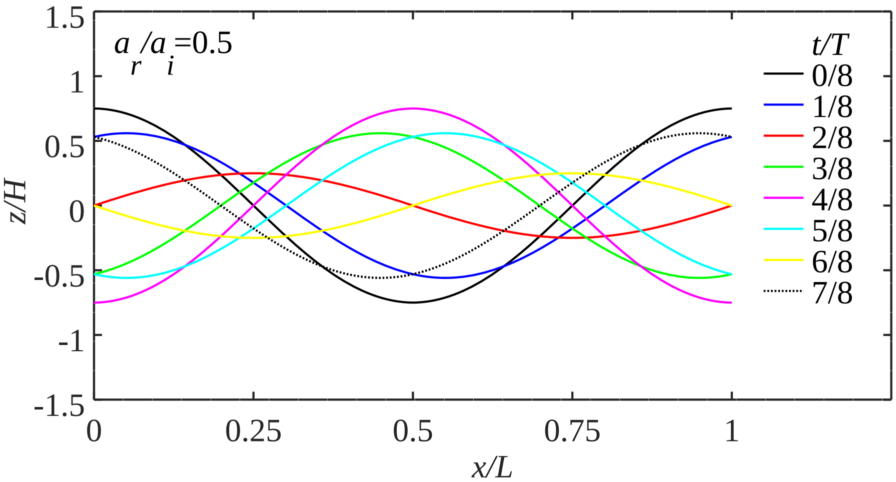


Figure 19: Partial standing wave

where the amplitude is the same as a. Superimposing the **Eq. (27)** and **Eq. (28)** gives the standing wave as follows:

$$\eta = \eta_i + \eta_r = a\cos(kx - \sigma t) + a\cos(kx + \sigma t) = 2a\cos kx \cos\sigma t \tag{29}$$

Equation (29) does not contain the progressive component such as $cos(kx-\sigma t)$ or $cos(kx+\sigma t)$. The free surface displacement simply changes up and down as the time passes. Therefore, it is called the standing wave. **Figure** 18 shows time histories of standing wave. It is observed that there are nodes at the 0.25 and 0.75 of x/L and loops at 0, 0.5 and 1 of x/L.

When the reflected wave has smaller amplitude than that of the incident wave, we observe the **partial standing waves** as shown in **Fig.** 19. The ratio of the incident wave amplitude to

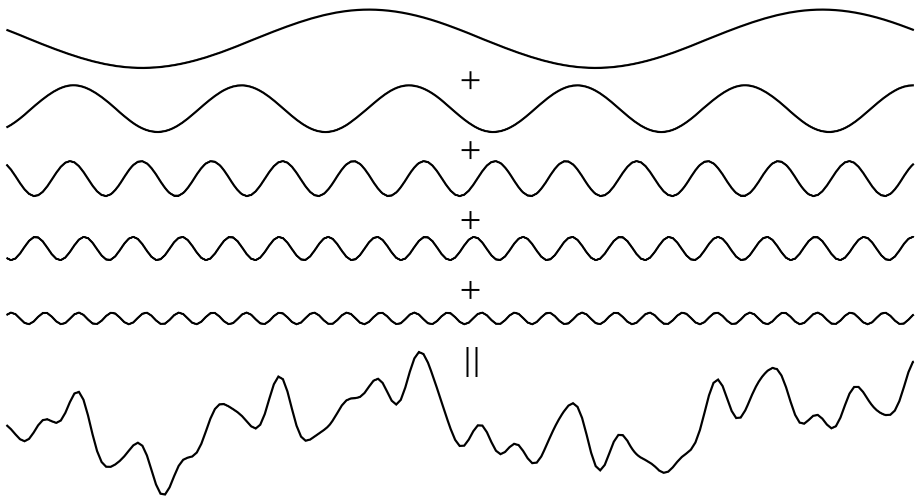


Figure 20: Superimposition of different frequency waves

that of the reflected wave is a_r/a_i=0.5. In this case, nodes are not as clear as that of the perfect standing wave.

Waves are such phenomena that superimposing of some components is applicable when we consider linear waves. Note that superimposing components is not valid when considering the nonlinear waves.

2.6 Irregular Waves

Waves in the sea are random waves or irregular waves of which wave heights, wave period vary in time and space. In addition to those variations, wave propagation direction also can broaden in 360 degrees.

First we consider irregular waves which propagates in the same direction. Free surface displacement of irregular waves η is described as the following equation when we assume that irregular waves consist of regular waves with different angular frequency.

$$\eta(x,t) = \sum_{n=1}^{\infty} a_n \cos(k_n x - \sigma_n t + \varepsilon_n) \tag{30}$$

where n is a wave component of irregular waves with the angular frequency σ_n and the wave number k_n. The relationship between σ_n and k_n is given by the dispersion relation of **Eq. (2)**. The symbol of a_n is an amplitude of a wave with the wave number k_n and ε_n is a phase shift of the wave component. **Figure** 20 shows a simplified figure of the superimposed time series of the irregular waves as a summation of regular waves which has different wave heights and periods.

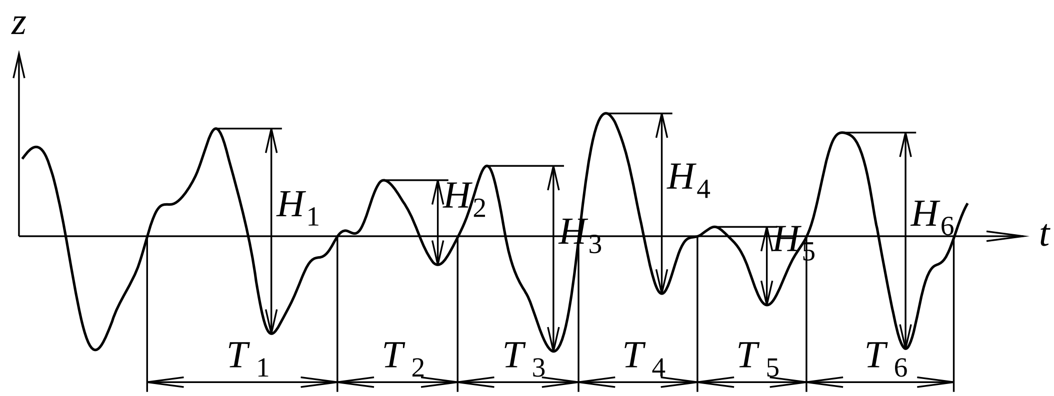


Figure 21: Wave train (wave-by-wave) analysis

The wave height and wave period of irregular waves change wave by wave. In order to handle the irregular wave for the engineering purpose, we need to define a representative wave height and period. **Figure** 21 shows wave train for a wave-by-wave analysis. Wave height and wave period are defined by the wave-by-wave method. Each wave period is defined as the one from a zero up crossing time to the next zero up crossing time. The each wave height is defined by the distance from the highest level to the lowest one. Other statistical quantities are commonly ascribed to sea states in the related literature and practice (Coastal Engineering Manual[8]).

Based on the definitions for this kind of individual wave, statistical quantities are obtained as follows: The mean of all the measured wave heights in the entire record analysed is called the mean wave height $\overline{H}$. The largest wave height in the record is the maximum wave height $H_{\max}$. The root-mean-square of all the measured wave heights is the rms wave height H_{rms}. The average height of the largest $1/n$ of all waves in the record is the $H_{1/n}$ where n = 10, 11, 12, 13,..., 99, 100 are common values. For instance, $H_{1/10}$ is the mean height of the highest one-tenth waves. In coastal projects, engineers are faced with designing for the maximum expected, the highest possible waves, or some other equivalent wave height. From one wave record measured at a point, these heights may be estimated by ordering waves from the largest to the smallest and assigning to them a number from 1 to N. The significant wave height $H_{1/3}$ or H_s will be the average of the first (highest) $N/3$ waves. The following wave heights and its wave periods are of importance for the coastal engineering aspect.

1. maximum wave ($H_{\max}$, $T_{\max}$)
2. 1/10 maximum wave ($H_{1/10}$, $T_{1/10}$)
3. 1/3 maximum wave or significant wave ($H_{1/3}$, $T_{1/3}$ or H_s, T_s)
4. mean wave ($\overline{H}$, $\overline{T}$)

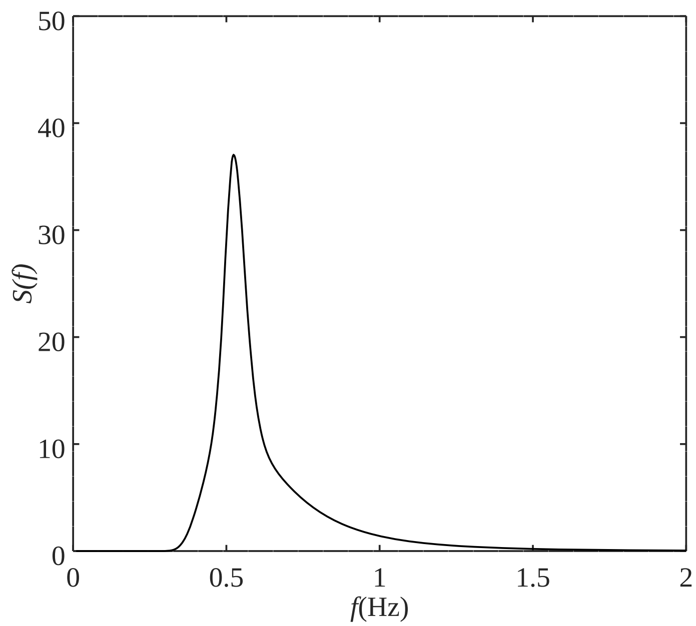


Figure 22: An example of power spectrum based on JONSWAP spectrum

When we expand the field from two-dimensional field to three-dimensional one, the free surface displacement is expressed by the following equation:

$$\eta(x, y, t) = \sum_{n=1}^{\infty} a_n \cos(k_n x \cos\theta + k_n y \sin\theta - \sigma_n t + \varepsilon_n) \tag{31}$$

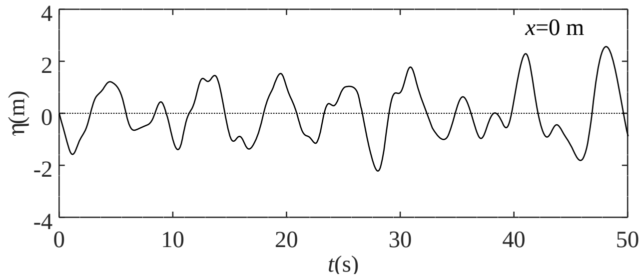
where $x - y$ is a plane field.

Free surface displacement η has a typical power spectrum such as the JONSWAP spectrum given by **Eq. (32)**.

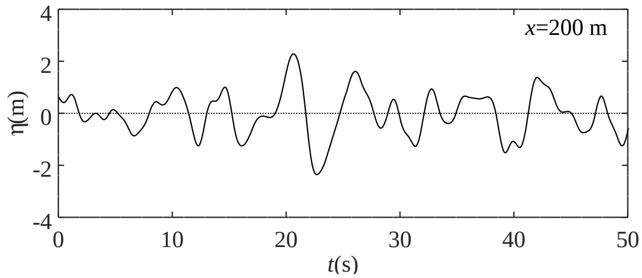
$$S(f) = \frac{\alpha g^2}{(2\pi)^4} f^{-5} \exp\left\{-\frac{5}{4}\left(\frac{f}{f_p}\right)^{-4}\right\} \gamma^{\exp\left\{\frac{-(f-f_p)}{2\pi^2 f_p^2}\right\}} \tag{32}$$

where f_p is a peak frequency and details are given, for example on the website[4]. **Figure** 22 shows an example of the power spectrum obtained using **Eq. (32)**.

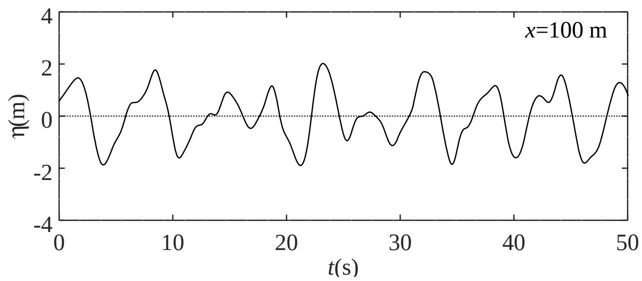
Time histories of irregular waves change at different places as shown in **Fig.** 23 because the components with different frequencies of irregular waves have different wave celerities. However, statistical values such as the wave energy, the wave height and wave period as well as the power spectrum remain constant.



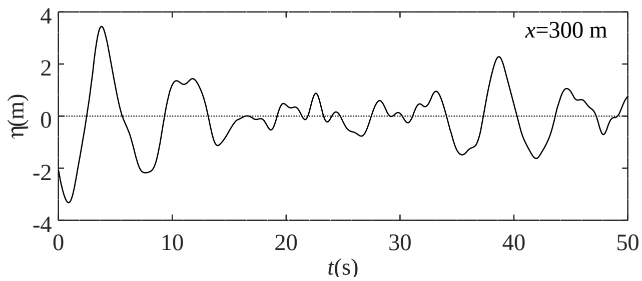
(a) x=0m



(c) x=200m



(b) x=100m



(d) x=300m

Figure 23: Time series of irregular waves at different locations while propagating.

2.7 Wave Refraction

As explained before, the wave celerity varies depending on the water depth. When waves propagate from one region to another region of which water depth differs, the waves are refracted. As is in the case of the refraction of the light, Snell's law is true for the water waves. **Figure** 24 shows an example of the wave propagation from the water of the water depth h_1 to that of h_2. The reflected waves are accompanied in this situation as shown with the dashed line. The relationship of the incident angle α_1 and the reflected angle α_2 is expressed by the following equation:

$$\frac{c_1}{\sin \alpha_1} = \frac{c_2}{\sin \alpha_2} \tag{33}$$

where c_1 and c_2 are wave celerities at the water depth h_1 and h_2, respectively.

The distance between the wave rays is given by **Eq. (34)**.

$$\frac{b_1}{\cos \alpha_1} = \frac{b_2}{\cos \alpha_2} \tag{34}$$

where b_1 and b_2 are the distance of wave rays at the water depth h_1 and h_2, respectively.

Figure 25 shows that the crest lines offshore about 45 degree from the east. However, those near the beach are parallel to the shoreline. The wave direction changes from the northwest offshore to the north nearshore. This change of the wave direction is caused by the wave reflection. As the wave celerity is a function of the water depth, the waves in the left in case of **Fig.** 25

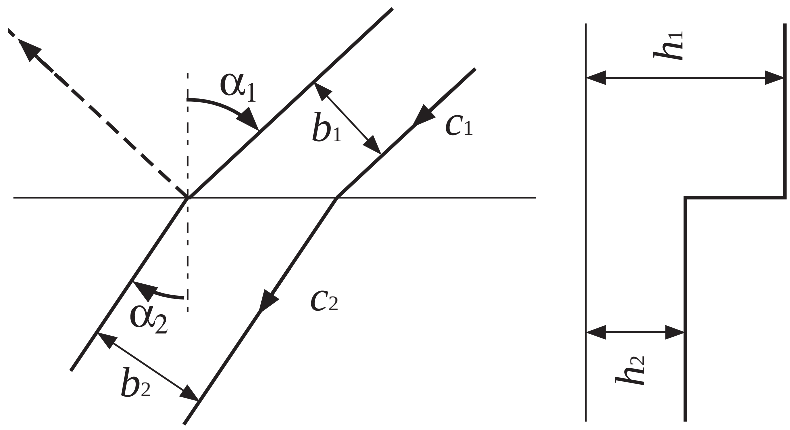


Figure 24: Snell's law for water waves

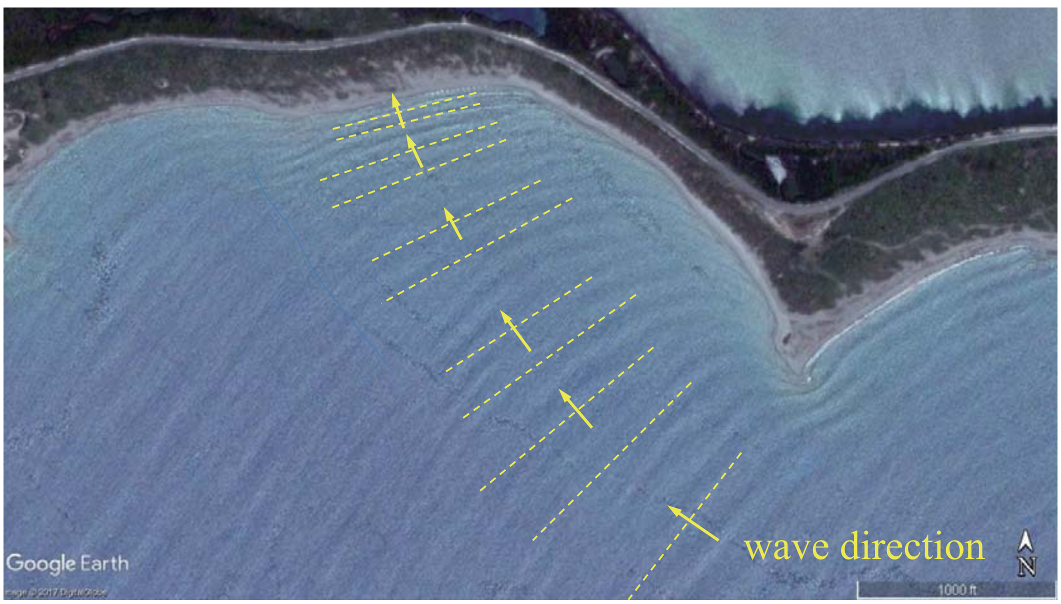


Figure 25: Wave refraction observed near Palisadoes

propagate faster than that in the right on the same wave crest. As a result, the waves turn clockwise then the wave crests are parallel to the shoreline.

Figure 26 shows the mechanism of wave refraction more generally than the Snell's law for water waves. When waves are approaching a shoreline on a uniform slope seabed, from a certain angle, the wave celerity along the wave crest line varies because of the difference of the water depth. Waves propagate faster in the deep water depth than in the shallow water. Consequently, the wave crest line makes the clockwise turn in the case of **Fig.** 26. As it continues, the wave crest becomes parallel to the shoreline.

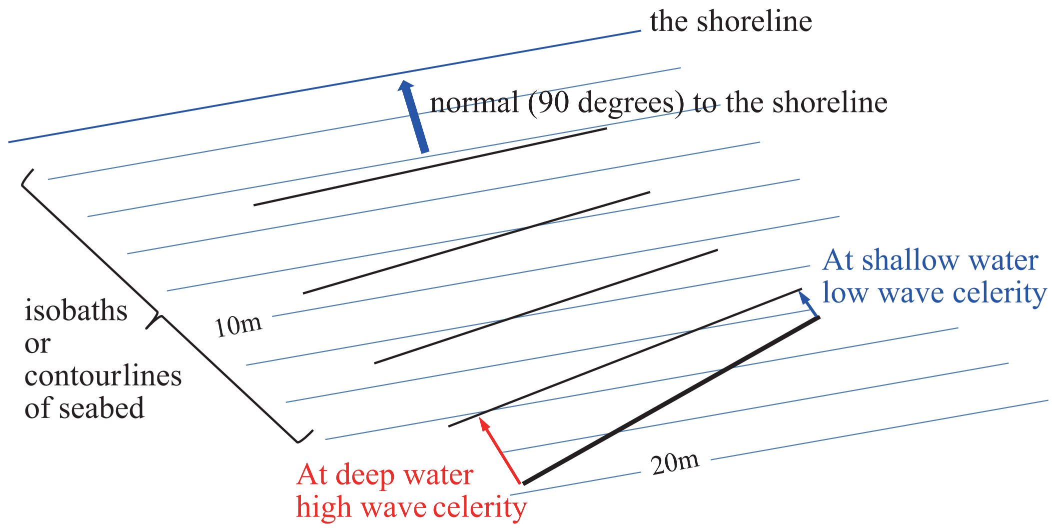


Figure 26: Mechanism of wave refraction on seabed with a uniform slope.

2.8 Wave Diffraction

Waves propagate behind structures. A structure makes a shadow when the sunlight is protected by the structure. However, an island cannot hinder the propagation of waves behind it due to the wave diffraction. **Figure** 27 shows an example of the wave diffraction due to a small island near Port Royal. In the eastern area of the island, waves propagate to the west. In the area between the island and the southern coastal line of Palisadoes, we can observe waves taking roundabout paths from both the right and left sides to the area behind the island.

When we consider waves and topography on a big scale, waves caused by a hurricane propagate into the Kingston Harbour behind Palisadoes due to the wave diffraction. This is depicted as shown in **Fig.** 28. When waves reach the tip of Palisadoes, cylindrical waves are generated and dispersed both outside and inside the Kingston Harbour. Progressive waves go through the harbour mouth. As waves propagate inside the harbour, the cylindrical waves decrease their wave height because the wave energy is dispersed through the periphery. Because a simple method is not available for evaluating the wave diffraction, we usually use a numerical simulation to estimate the wave height of the diffracted waves.

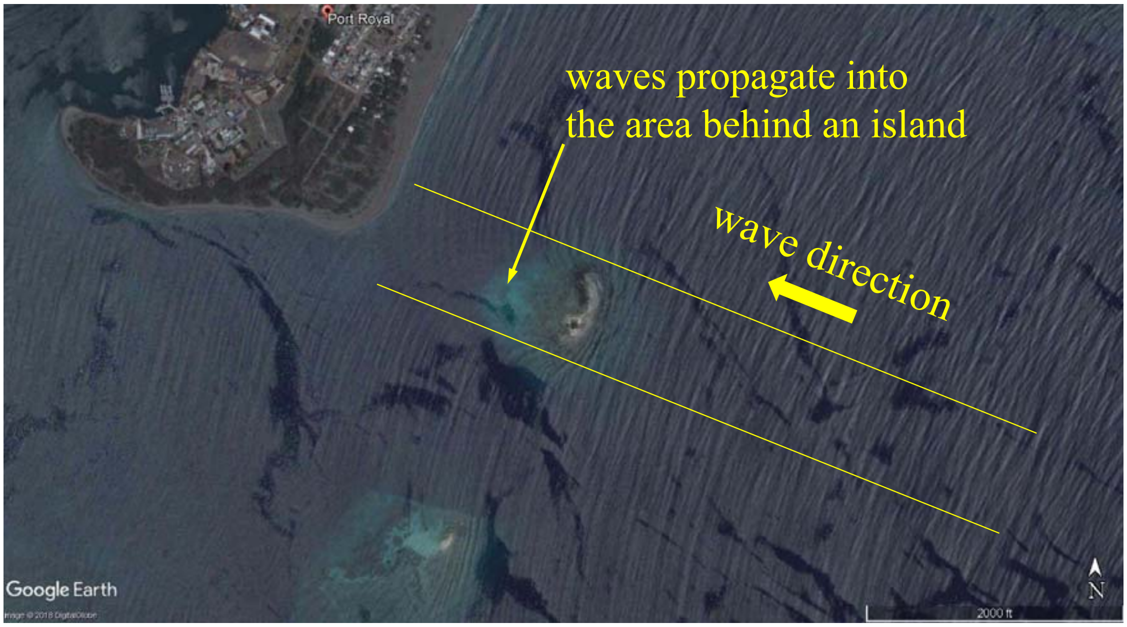


Figure 27: Wave diffraction obeserved behind an island near Port Royal

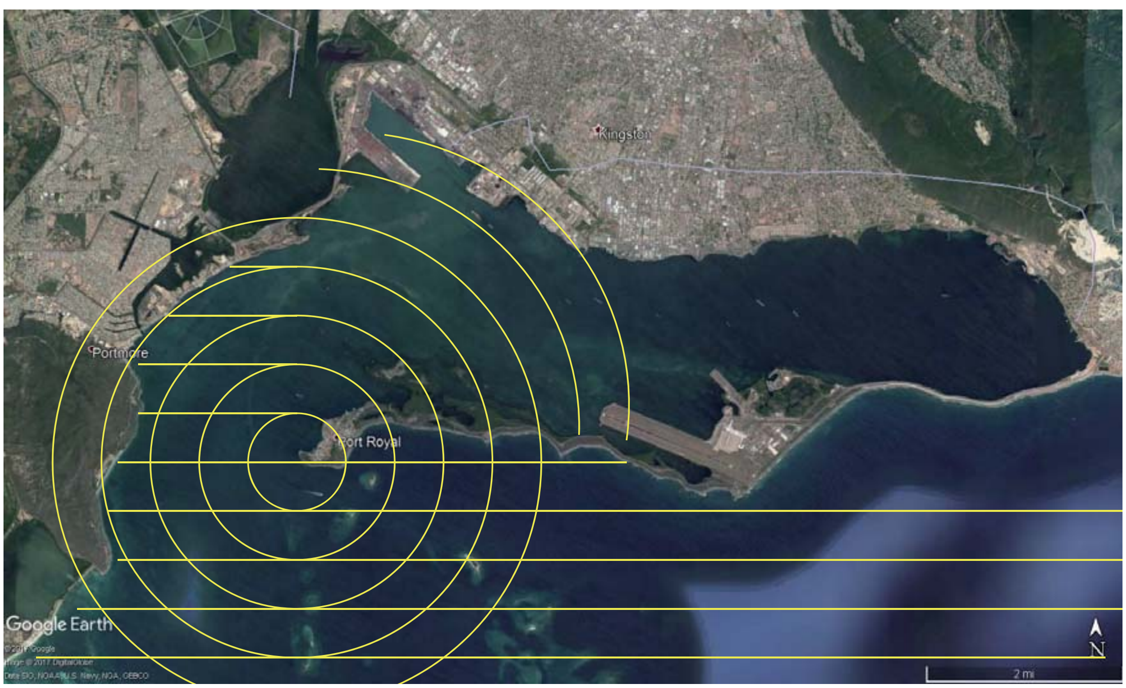


Figure 28: Schematic drawing of wave diffraction in Kingston Harbour.

3 Nearshore Currents

In the sea, several types of unidirectional currents exist due to different causes. Winds cause drift current, for example. Waves also cause unidirectional currents in the sea. Some of currents accompanied by waves are described in this section.

3.1 Mass Transport due to Waves

The mass of the water is transported due to wave motion from offshore to onshore. It is called the **mass transport**. It is noted that the mass transport takes place due to the deepwater waves offshore and to nearshore. The description of the mass transport is included in this section of nearshore current because the appropriate section is not available in this textbook.

As described about the water particle motion of waves in the subsection 2.1.4, closed orbit paths are made under the free surface as shown in **Fig.** 8. This is the result based on the linear wave theory. However, to be exact those orbits do not close after one wave period. When the water particles move under the wave crest, they follow the velocity at higher water level in the onshore direction. On the other hand, when they move under the wave trough, they follow the velocity at lower level in the offshore direction. There is a difference of velocity between onshore and offshore velocities. It does not make an exact closing orbit resulting in the small progress in the direction of the wave propagation. The mass transport velocity is given by the following equation, **Eq. (35)**.

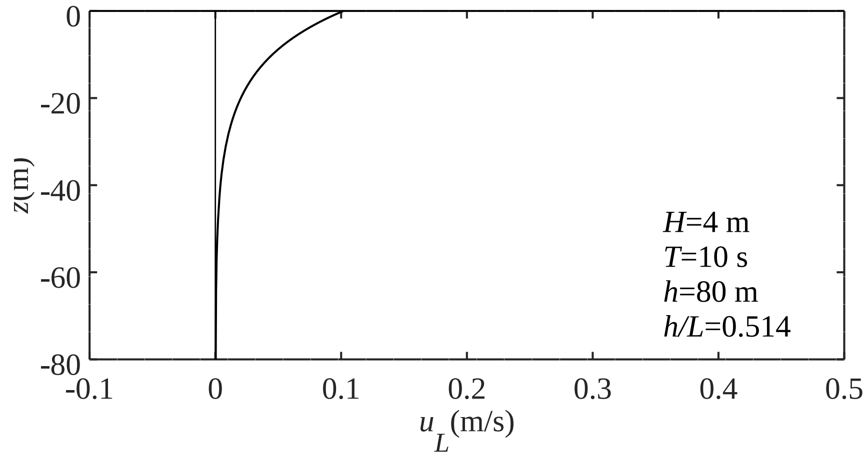
$$\overline{u_L} = \frac{1}{2}a^2ck^2\frac{\coth 2k(h+z)}{\sinh^2 kh} \tag{35}$$

Figure 29 shows examples of the profiles of the mass transport velocities of the deep water wave, shallow water wave and long wave. These mass transport velocities are quite small such as 0.1m/s. It is approximately one tenth of order of that of the velocity of water particle of wave motion.

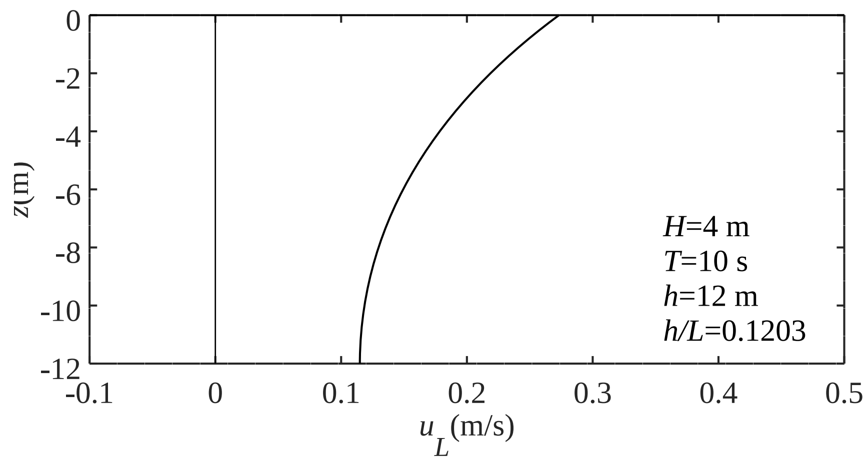
The amount of water transported onshore M_x is obtained by integrating **Eq. (35)** from the sea bottom to the free surface, where $\overline{a}$ means the time average of a certain physical quantity a over one wave period.

$$M_x = \overline{\int_0^\eta \rho u \mathrm{d}z} = \frac{1}{2}\rho\sigma a^2 \coth kh \tag{36}$$

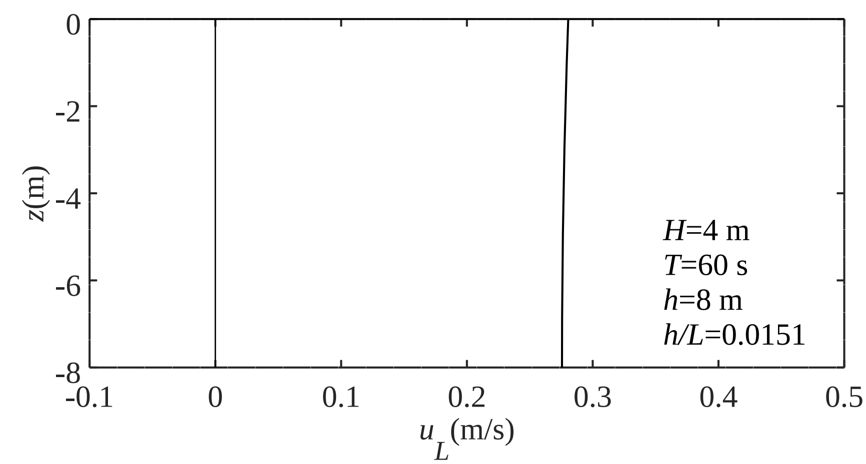
The amount of the mass transport is also small as a current. However, it is noted that it causes the rip current nearshore when a wave direction is normal to the shoreline.



(a) Deep water wave



(b) Shallow water wave



(c) Long wave

Figure 29: Mass transport velocity profiles.

3.2 Bore due to Breaking Waves

Breaking waves generates currents in the surf zone. After waves break, they behave like they are no longer waves but currents. Mass of sea water is transported onshore because the motion changes from an orbital one to an unidirectional one. The bore-like current is observed near the sea surface in the surf zone as shown in **Fig.** 30.

Figure 30: Bore after wave breaking

3.3 Undertow

The amount of the water transported due to the mass transport and the bore due to the wave breaking should be reversed offshore. Otherwise, the water could be accumulated near the shoreline, which is not sustainable. In case of beach with a relatively steep slope, an offshore-directed current in the middle of water depth or on the sea bottom is generated. This current is almost uniform along the shoreline. This is called the **undertow**.

3.4 Rip Current

So far currents are two dimensional phenomena seeing the flow on the vertical plane. Rip current and longshore current are observed on a horizontal plane.

Figure 31 shows a schematic view of the rip current and waves. When the wave direction is normal to the coastal line, the rip currents are generated with a certain interval in the direction of shoreline. The distance of the interval is for example, several hundreds meters long. This is caused by the mass transport due to waves. The sea water is transported to the shore and it

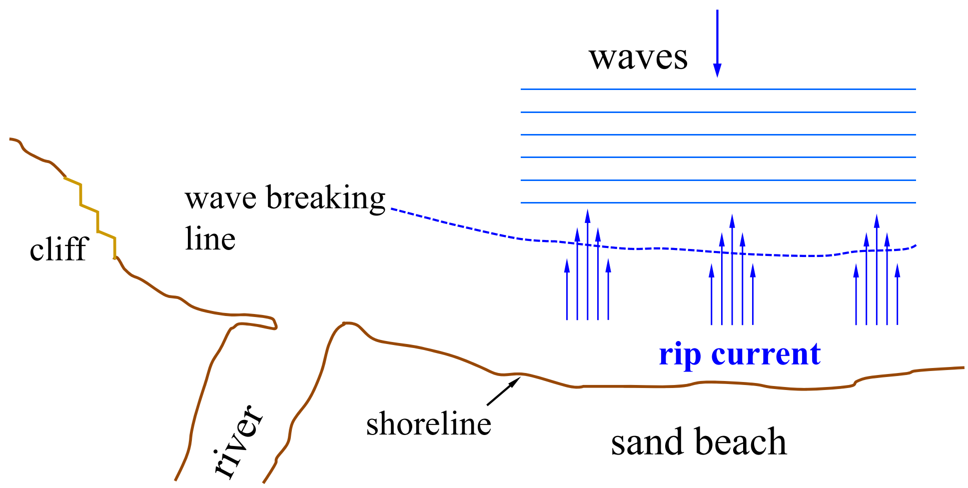


Figure 31: Rip current

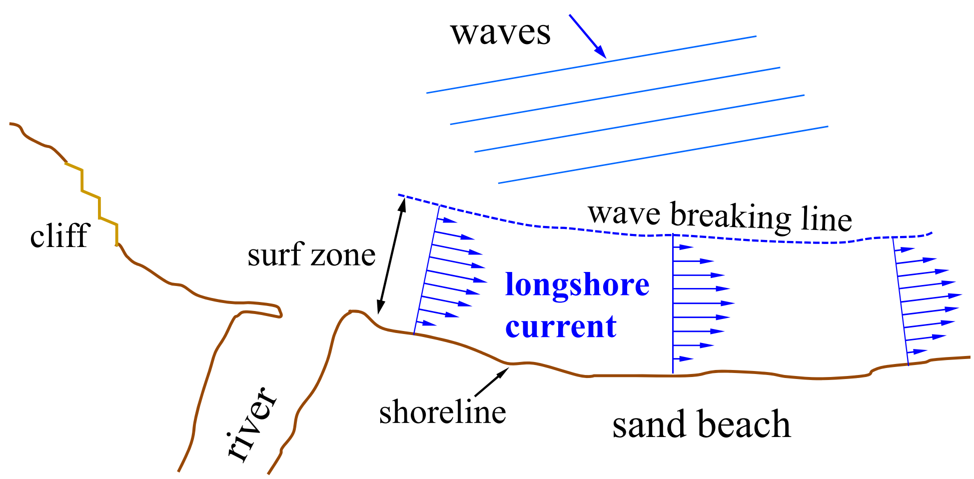


Figure 32: Longshore current

should be reversed offshore resulting in a current. This is called the **rip current**. The current is not uniform in space but it is concentrated in a narrow width such as 10 m or 20m wide. The velocity of the rip current can be more than 1 m/s. Therefore, channels in the seabed are formed along the rip currents.

3.5 Longshore Current

When the wave direction is oblique to the shoreline, breaking waves cause a uni-directional current along the shoreline as shown in **Fig.** 32. It is called the **longshore current**. Momentum due to the breaking wave has a longshore component which causes the longshore current. Velocity of the longshore current is not necessarily uniform alongshore. It can cause beach topography change along the shoreline.

4 Coastal Sediment Transport

Sediment transportation is usually classified into two mechanisms, that is, **bed load** and **suspended load**. The sediment transport nearshore takes place in the on-offshore direction and in the longshore direction. The former is called the **cross-shore sediment transport** and is mainly caused by waves. The latter one is called the **longshore sediment transport** and is induced by the longshore current in the surf zone.

4.1 Bed Load

On the seabed in a relatively deep water, sediment moves in a mode of the **bed load**. Sand particle or gravel moves along the seabed in the mode of rolling, sliding and/or saltating according to the wave motion back and forth. The asymmetry of the orbital motion of nonlinear waves results in net sediment transport.

4.2 Suspended Load

Sediment is transported in a mode of the **suspended load** once it is uplifted into the body of water, for example, by breaking waves nearshore. Then it is transported with currents, such as bore or longshore currents due to breaking waves. The suspended load is kept in the fluid by the turbulence of the flow.

4.3 Cross-shore Beach Change

Figure 33 shows the shoreline change patterns due to the cross-shore (on-off shore) sediment transport (Sunamura and Horikawa,1974[5]). The cross-shore sediment transport is mainly caused by wave motions. There are three types of beach change called Type I, Type II and Type III. Type I is erosion and Type II and III are accretion. These are caused by the sand movements as shown in **Fig.** 33. The shoreline change is accompanied by the topography change in the nearshore zone. Especially the wave breaking plays an important role in the beach change. Arrows in **Fig.** 33 show the possible direction of the net sediment transport. In the case of TYPE I, the sediment was transported from the shoreline to the offshore area resulting in the bar. In the case of TYPE III, the sediment around the breaking point is transported onshore resulting in the accretion near the initial shoreline. In the case of TYPE II, the sediment was transported both to the onshore and to offshore sides.

The criteria between erosion and accretion on **Fig.** 33 is given by Sunamura and Horikawa (1974)[5]. **Equation** (37) gives the criterion whether shoreline is erosion or accretion depending

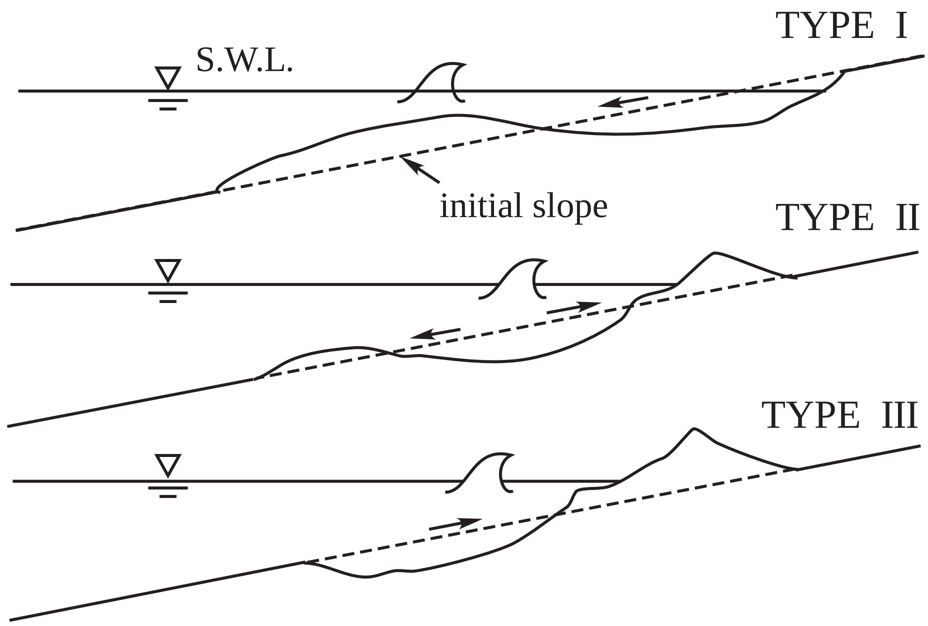


Figure 33: Shoreline change patterns (Sunamura and Horikawa, 1974)[5]

on the wave conditions, beach slope and sediment.

$$\frac{H_0}{L_0} = C_s(\tan\beta)^{-0.27}\left(\frac{d}{L_0}\right)^{0.67} \tag{37}$$

where H_0 is the deepwater wave height, L_0 the deepwater wavelength, β the sea bottom slope, and d the mean diameter of the sediment. The coefficient denoted by C_s is the threshold to determine whether the accretion takes place or the erosion does. The value of this coefficient is obtained by the field data on beach change and is equal to 18. This value is confirmed by full-size experiments as shown in **Fig.** 34 (Kajima et al., 1982[6]) According to **Eq. (37)** and **Fig.** 34, when the deepwater wave height H_0 increases, shoreline tends to retreat. While the wavelength L_0 or the wave period T increases, shoreline tends to accretion insignificantly. As the seabed slope $\tan\beta$ increases, the shoreline tends to be eroded. As the mean diameter d of sediment increases, the shoreline get started accretion, remarkably. Roughly speaking, when the wave steepness H_0/L_0 is small, accretion takes place.

In other words, after beaches are eroded by high waves, they can be recovered by small waves which frequently surge. This phenomena are observed as seasonal events at beaches in Japan. Beaches are eroded in winter but they are recovered through other seasons. Consequently, beaches remain almost unchanged.

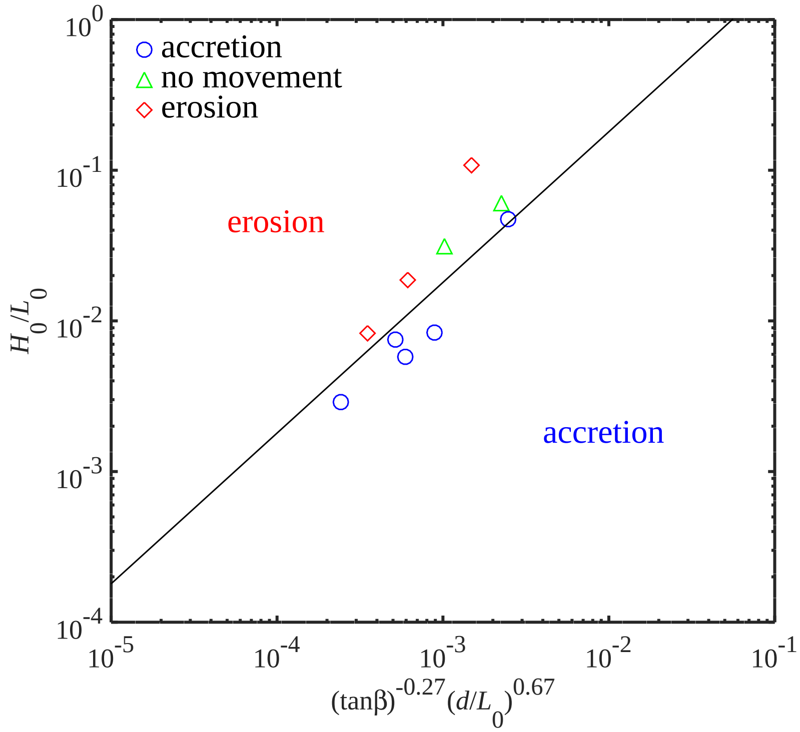


Figure 34: Comparison with experimental results (Kajima et al., 1982[6])

4.4 Longshore Beach Change

Figure 35 shows a schematic view on the beach change due to the longshore current. In the surf zone which is defined as the region between the wave breaking line and the shoreline, the longshore current runs as explained in the subsection 3.5. The longshore current transports sediment along the shoreline. The sediment is supplied from the seabed due to the breaking waves and also through the river mouth into the sea. Cliff erosion due to waves is also one of the sediments resources.

Figure 36 shows an example of the longshore sediment transport observed along the coastline near Habourview. The river mouth of the Hope River is seen on the left side on the satellite photo. Sediment clouds were extended to the outside of the surf zone.

If the longshore sediment transport rate changes along the shoreline, beach change takes place. When the longshore sediment transport rate on the upstream side is lower than that on the downstream side, the beach suffers from erosion. If the longshore sediment transport rate on the upstream side is higher than that on the downstream side, the beach expands offshore, that means the accretion.

Even if the longshore sediment transport occurs and its amount is uniform along the shore,

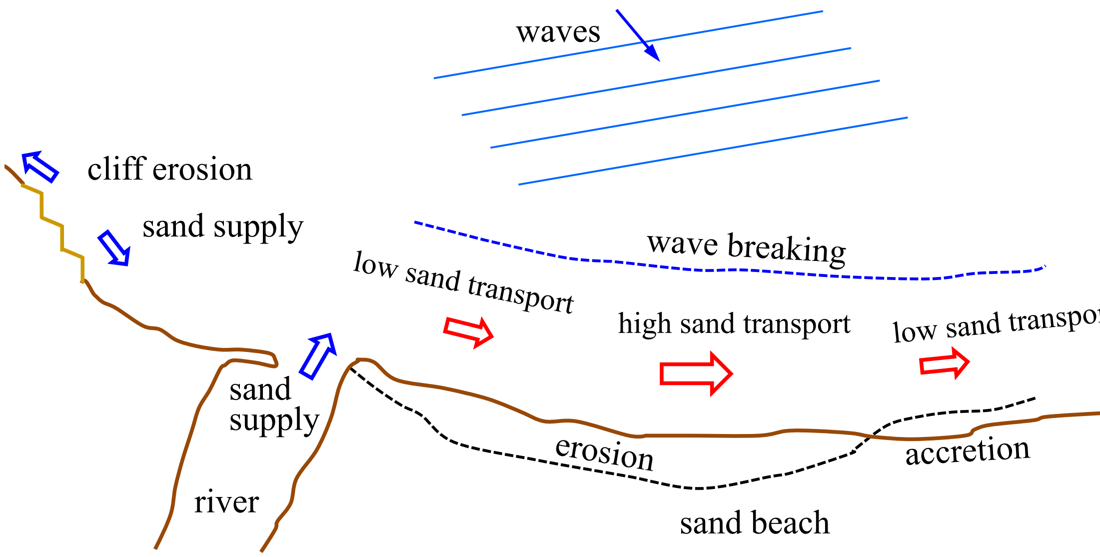


Figure 35: Beach change due to longshore sediment transportation.

Figure 36: Longshore sand transport observed along the coast near Habourview.

beach does not necessarily suffer from erosion and accretion. In other words, when the amount of the sediment transported at a certain position is the same as that of the sediment replenished, there is no change in the beach topography.

The continuity equation of the sediment transport is given by the following equation.

$$\frac{\partial z_b}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0 \tag{38}$$

where z_b is the elevation of seabed, q_x and q_y are the sediment transport rate to the cross-shore and that along the shoreline,respectively. When q_x or q_y changes in space, the elevation of the seabed changes.

4.5 Equilibrium of Beaches

When a beach is attacked by big waves such as waves generated by a hurricane, it will usually be suffering from erosion. Sometimes sediment could be transported onto the beach piling up.

On the other hand, when sand on the beach does not move at all, we call it **static equilibrium**. Even when beaches seems to remain stable, sand can be transported by waves and/or currents. We call it **dynamic equilibrium**. However, waves are big enough to move sand, beaches are usually dynamically equilibrium or suffering from erosion. When we consider the possibility of beach erosion, we should focus on the resources of sediment which replenishes sediment to the beach. In the case of beaches facing open sea, a river is one of the resources. In the case of coral reef beaches, carbonate should be replenished from coral reef itself.

5 Sediment Resources

Jamaican beaches consist of two kinds of sediments such as silicate or carbonate. The former is transported through rivers to beaches. On the other hand, the latter is produced by coral reef organisms such as coral, algae and other seagrasses.

High waves easily transport sediment resulting in beach erosion when the sediment is fine. Therefore, considering restoration of beach, we should focus on two factors such as wave conditions and resources of the sediment. Once a beach has been eroded by hurricane-generated waves, it takes time to recover. In order to restore the beach, it has to be replenished with sediment. It is of importance to find out where the sediment comes from. If the resource of sediment is not available, the beach will never be recovered. In other words, obstacles to beach restoration are river mouth blocking which prevents from replenishing sediment to beaches and unhealthy coral reef which cannot produce sediment for white beaches.

5.1 Rivers as Transportation Path of Sediment

Beaches facing open seas consisting of silicate-based sediment are of types of sand beach and gravel beach. Sand beach is seen in Annotto Bay and gravel beaches in Port Royal and Morant Bay. The gravel size in the northwest beach of Buff Bay is much greater than those in Port Royal and Morant Bay. That difference may be caused by difference of sediment resource and the length of the rivers. Short river cannot change stone to sand due to the lack of the abrasive time for stones.

This section introduces two rivers which transport sediment. One is the Yallahs River which runs to the south coast in Jamaica. The other one is the Wag Water River to the north coast.

Fgiure 37 shows the locations where photos were taken with yellow balloons along the Yallahs River. The Yallahs River is 36.9 km long. **Fgiure** 38 shows locations where photos were taken with yellow balloons along the Wag Water River. The Wag Water River is 36.2 km long.

Fgiure 39 shows photos of the Yallahs River from upstream to downstream. The river upstream is full of vegetation which prevents river water from running downstream. When it rains heavily, flooding takes place and prevents residents from passing the bridge. There are mining sites at the midstream and downstream of the Yallahs River. From this fact, the Yallahs River is considered as a path which transports a large amount of sediment. The issue is whether or not enough sediment reaches the coast to replenish the southern beaches.

Fgiure 40 shows that the river bed is covered with rubbles. There is a mining site near the river mouth where a heavy machine was used to mine sediment from the river as shown in **Fig.** 40(f). The Wag Water River also transports a large quantity of sediment.

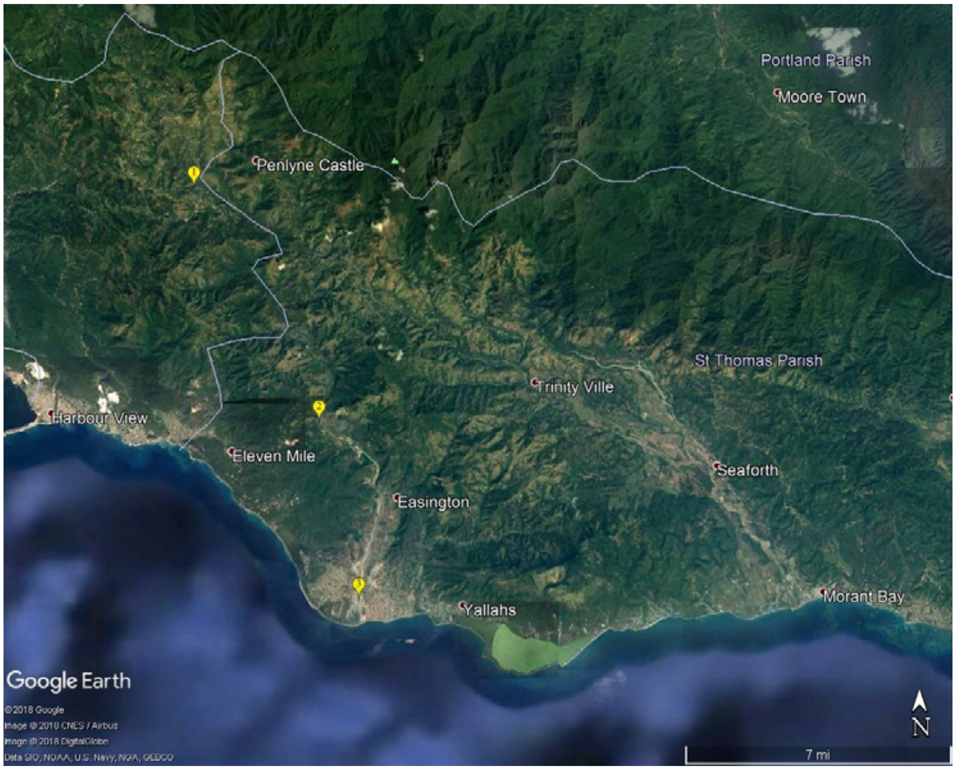


Figure 37: Locations of photos taken along the Yallahs river.

Figure 38: Locations of photos taken along the Wag Water river.

(a) Upstream of location no.1 in **Fig.** 37

(b) At aggregating site in midstream of location no.2 in **Fig.** 37

(c) From the Yallahs Bridge of location no.3 in **Fig.** 37

Figure 39: The Yallahs River

(a) Location of no.1 in **Fig.** 38

(b) Location of no.2 in **Fig.** 38

(c) Location of no.3 in **Fig.** 38

(d) Location of no.4 in **Fig.** 38

(e) Location of no.5 in **Fig.** 38

(f) Location of no.6 in **Fig.** 38

Figure 40: The Wag Water River

5.2 Coral Reef Organisms

Jamaica is surrounded by fringing coral reefs. A cross section of a fringing coral reef is shown in **Fig.** 41 and a plane view is in **Fig.** 42, respectively. Waves break at fore reef zone and dissipate most of the wave energy. Therefore, in the shallow water zone called lagoon zone, waves with small wave heights propagate. Tourism benefit from the phenomena.

In the case of coral reef beach, resource of sediment is coral reef organisms. Most reefs produce far more sediment than they can accommodate internally. The excess is carried throughout the reef system and any remaining surplus transported out of the reef environment and even into the deep sea (Dudley, 2003[7]).

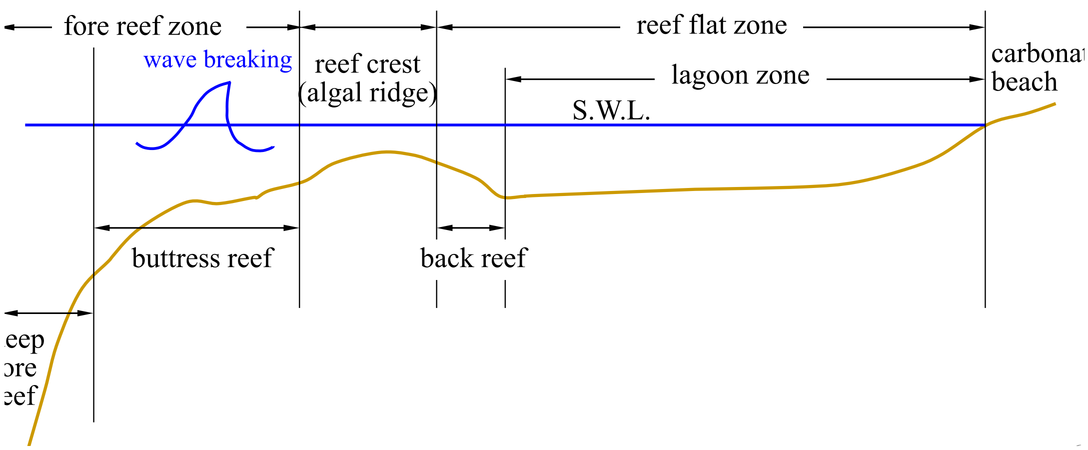


Figure 41: Cross section of coral reef

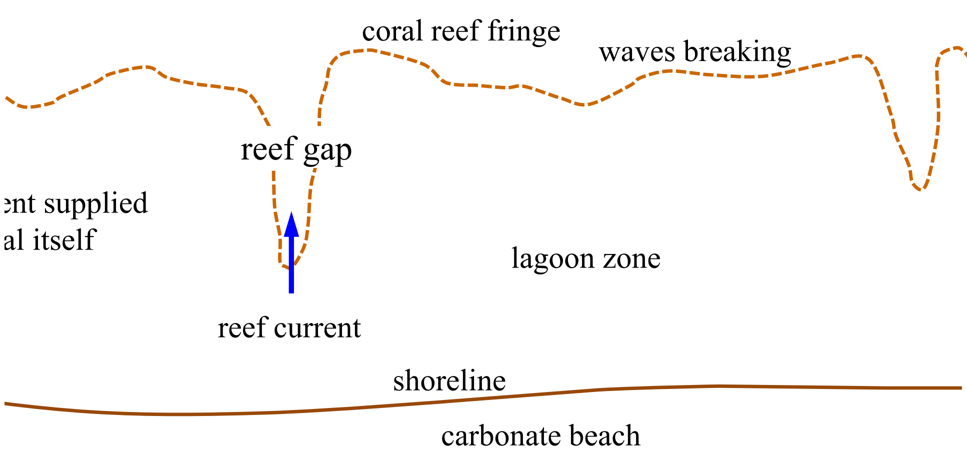


Figure 42: Plane view of coral reef

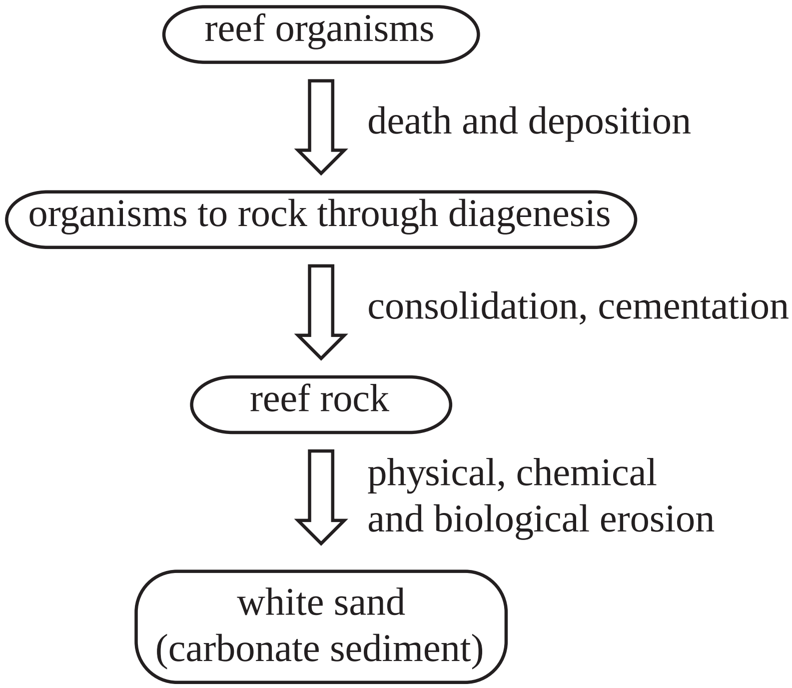


Figure 43: Process from coral reef organisms to sediment[7].

The process of sediment production is summarized in **Fig.** 43. After reef organisms died and settled on the seabed, they become reef rock after taking diagenesis process such as consolidation and cementation. Then reef rocks are eroded away physically, chemically and biologically. Waves erode rocks and parrot fish eats dead coral. Eventually it becomes white sand. Beach rock has been reported to have formed within two years at Dry Tortugas in the Caribbean and in only six months at a site on the Great Barrier Reef[7]. It seems to take time from death of reef organisms to white sand.

A photo shown in **Fig.** 44 shows Negril Environmental Protection Area. It advocates protecting the environment in Negril. One of the five photos insists that 'coral reefs help to replenish white sand on the beach'.

Figure 44: Panel of Negril Environmental Protection Area

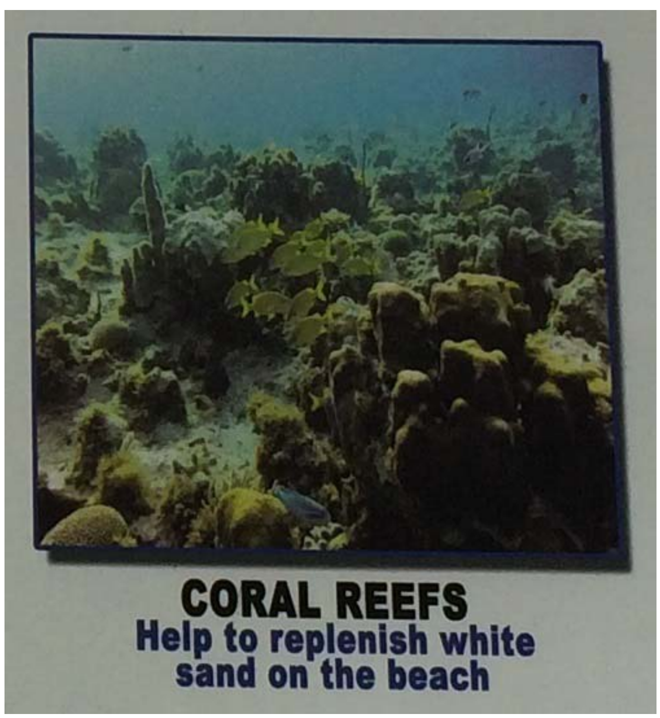


Figure 45: Photo to advocate replenishing white sand by coral reefs.

6 Examples of Beach Erosion and Formation in Jamaica

6.1 Beach Erosion in Annotto Bay

Figure 46 shows the beach in town centre of Annotto Bay. **Figure** 46(a) shows the coast behind Annotto Bay Clinic. One of the residents explained that there used to be wide beach several years ago. After being eroded, it has not been recovered. Annotto Bay Clinic suffers from wave action when the wave height is large. Especially, waves reach the foundation of building and erode the base ground. **Figure** 47 shows beach between the town centre and the Wag Water river mouth. The long beach is still in place although it is not wide.

(a) Facing west

(b) Facing east

Figure 46: Beach in town centre of Annotto Bay

(a) Facing west

(b) Facing east

Figure 47: Beach facing Annotto Bay

(a) Sediment discharge from the Wag Water River 18th of November, 2005)

(b) The Wag Water River mouth blocking(27th of January, 2017)

Figure 48: Coastline along Annotto Bay and Wag Water River mouth

Figure 48 shows a comparison of satellite photos of coastline facing Annotto Bay with river mouth of the Wag Water River between November 2005 and January 2017. Although the Wag Water River has enough capacity to supply sediment to Annotto Bay, which was explained by the fact that an aggregate mining operation is sited near the river mouth, it seems that sediment is prevented from being discharged by the river mouth blocking as shown in **Fig.** 48(b). Sand dune or sandbar at a river mouth is usually flushed when the river floods. Otherwise, river training is required to keep river mouths open resulting in sediment supply to coast.

6.2 Beach Erosion in Alligator Pond

The beach in Alligator Pond, especially in the Little Ochie area shown in **Fig.** 49, also has been eroded resulting in some damages such as the collapse of a commercial building, the exposure of the foundation of some houses to waves as shown in **Fig.** 50. The beach erosion has caused the drastic reduction of the beach itself and some business spaces[9].

(a) Facing east

(b) Facing west

Figure 49: Beach in Alligator Pond

(a) Collapse of a commercial building.

(b) Exposure of the foundation to waves.

Figure 50: Damages due to beach erosion in Alligator Pond

In order to find out the causes of beach erosion in Alligator Pond, we have to see a wide range of the coastline. It is said that along the southern coast of Jamaica, the dominant wave direction as well as the wind direction is mainly to the westward. The longshore sediment is also expected to be transported from the east to the west. Therefore, beach erosion in Alligator Pond could be

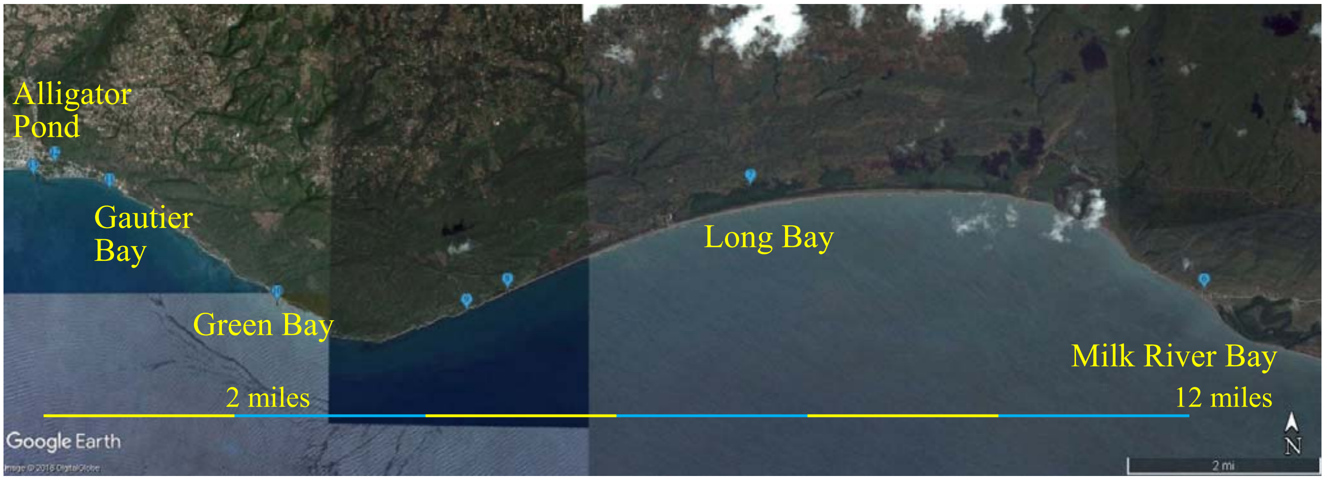


Figure 51: Coastline from Milk River to Alligator Pond

caused by the lack of sediment transport from the east coast. There are no big rivers on the east side close to this region.

Figure 51 shows a satellite photo of coastline from Milk River Bay to Alligator Pond Bay by Google Earth. The Milk River runs into the sea about 12 miles(19.2 km) east from Alligator Pond. There are a couple of rivers discharging into the sea between Milk River Bay and Long Bay, which are smaller than the Milk River. Blue balloons on the photo show the places where the author visited and took photos of coastline.

Figure 52 shows comparison of coastlines near the Milk River mouth between in 2003 and 2017. There are two beaches indicated by ovals which were formed between 2003 and 2017 as shown in **Fig.** 52(b). It is said that the dominant wind direction along the southern coast of Jamaica is from east to west. Therefore, it is understandable that two beaches on the west side of the Milk River were formed. It is interesting that the beach at the river mouth of the Milk River extends to the east. Some external driving force directing the east transported the sediment and the east side beach was formed. It seems the Milk River carried enough sediment transport from the upstream. However, **Fig.** 52(b) in 2017 shows the river mouth blocking by dune and a small gap of the stream. That means enough sediment was not supplied from the Milk River to the beaches nearby.

Moving to the west, **Fig.** 53 shows a part of the beach facing the Long Bay in Manchester Parish. There are a couple of rivers discharging into the Long Bay, which are smaller than the Milk River. The locations of the shoreline on the satellite photos include some uncertainty because tide levels are not exactly the same at those times when the satellite photos were taken. According to the three sequential satellite photos, however, it can be recognised that the beach along Long

(a) Coastline on 5th of February, 2003

(b) Coastline on 17th of February, 2017

Figure 52: Comparison of coastlines nera the Milk River mouth

Bay gradually developed between 2003 and 2017.

Moving to the west further from the Long Bay beyond. **Figure** 54 shows the grey sand beach on the rocky bed. The coastline is facing Green Bay. The beach is quite small. Vegetation along the coastline can be exposed to waves. Some trees on the beach are dead. Sand barely remains

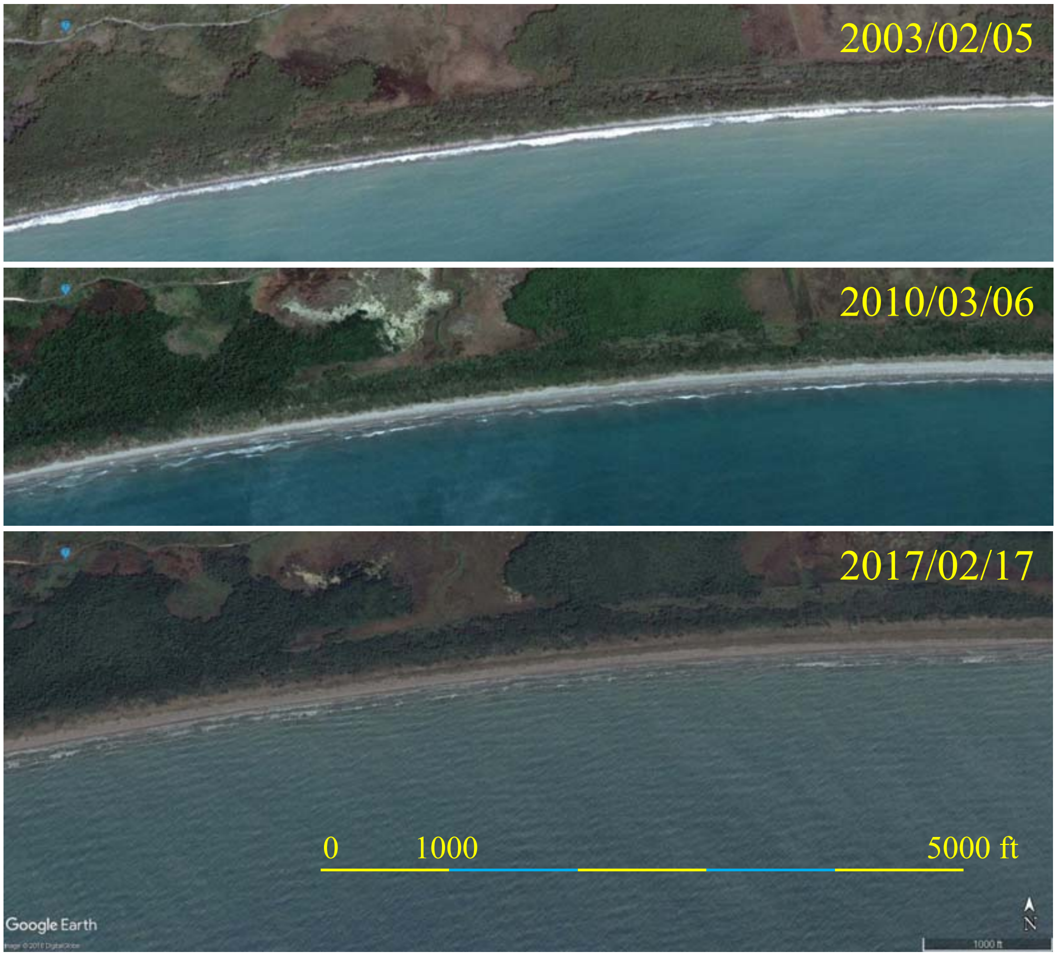


Figure 53: Beach change of Long Bay, Manchester

there. **Figure** 55 shows the comparison of coastlines between in 2001 and in 2009. It is recognised that this beach is sited close to the foot of mountain. The road seems to be constructed by cutting a part of the mountain slope. In 2001, there used to be a narrow beach along the road. However, the comparison indicates this beach was severely eroded between 2001 and 2009.

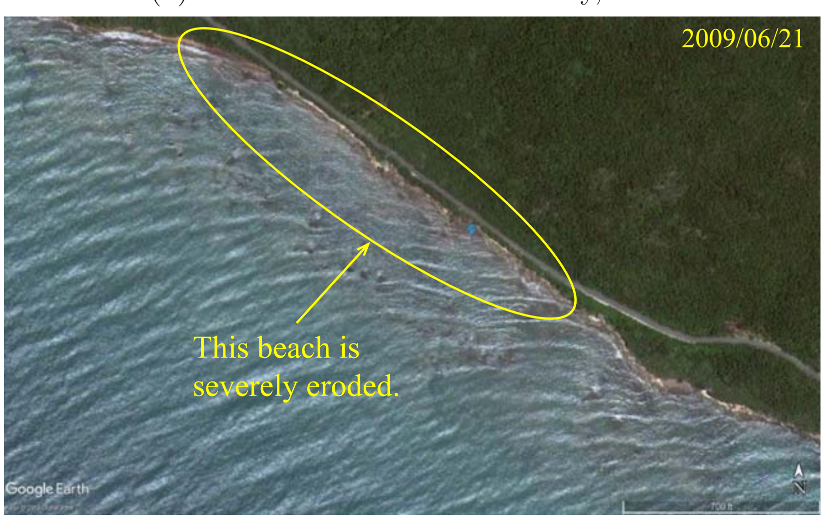
Figure 56 shows coastline facing Gautier Bay which is between Green Bay and Alligator Pond. There is a smaller beach than that in Green Bay The rocky coast is facing the sea.

Both **Fig.** 55 and **Fig.** 57 show beach on rocky bed. These two sites are located at the foot of the mountain. Rocky layer exists being covered with thick sand layer which is confirmed by the existence of dune in this area. **Figure** 57 shows the comparison of coastline between in 2004 and 2010. It is conformed that there was a beach in 2004. According to the satellite photo taken in 2010, the beach in 2004 was eroded and rocky bed appeared in 2010. In the west side of the rocky bed, the beach extended compared with that area in 2004.

(a) Facing east (b) Facing west

Figure 54: Beach facing Green Bay

(a) Coastline on 26th of January, 2001



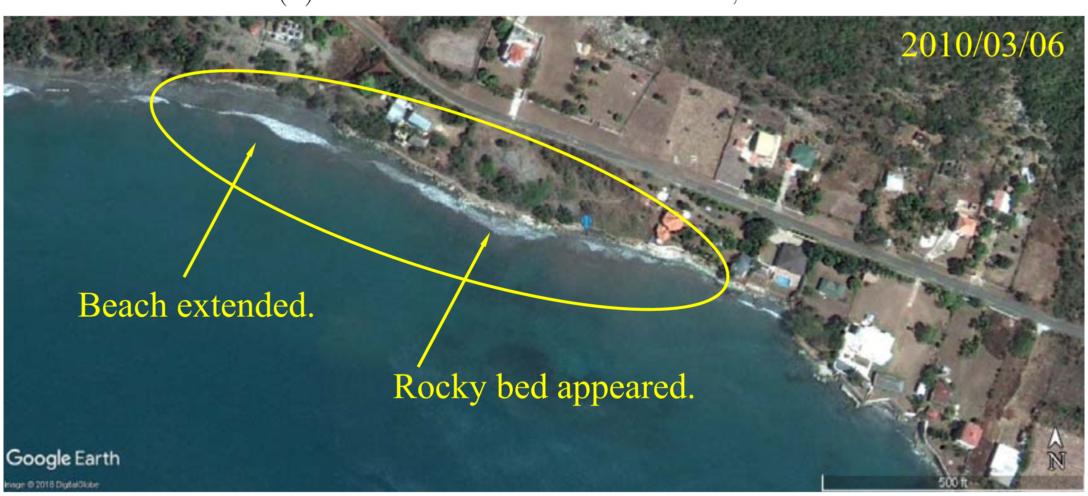
(b) Coastline on 21st of June, 2009

Figure 55: Comparison of coastlines facing Green Bay

(a) Facing east (b) Facing west

Figure 56: Beach facing Gautier Bay on east side of Alligator Pond

(a) Coastline on 22nd of Noveber, 2004



(b) Coastline on 6th of March, 2010

Figure 57: Comparison of coastlines facing Gautier Bay

Figure 58: Beach erosion from east to west around Alliagtor Pond.

Figure 58 shows the coastal area from Alligator Pond to the location with the blue balloon number 11 as shown in **Fig.** 56 where both rocky bed and dune coexist. An arrow on **Fig.** 58 indicates the place where a dune was observed as shown by the inserted photo. The dune is several meters high and exists at the foot of the mountain. An area in the yellow curve is covered with sand, from beach sand to dune at the foot of mountain.

Both beaches in Alligator Pond and in the eastern area are suffering from erosion. Sand could be moved from here to the west. However, beaches may not be replenished by sand from the farther east. It seems that this area is eroded from the east to the west as the yellow curve makes the shape. This is a warning sign that beach erosion could be in progress in the wide range of this coast. Considering the long time span of an order of geological scale, there could be concern that this area will disappear to the foot of the mountain as we can see the erosion on the beach facing the Green Bay and the Gautier Bay. The Milk River mouth is almost closed according to the satellite photo shown in **Fig.** 52(b). In order to replenish sediment from the Milk River to these beaches river training is required.

6.3 Beach Formation in Rocky Point

The beach in Rocky Point has a history to be formed by longshore currents and groins. It gives us very useful information about beach formation.

Figures 59 shows coastline of Rocky Point. The two coastlines denoted by the green and blue dashed lines are connected unnaturally with a crossshore shift indicated by the red dashed line. **Figure** 60 shows the beach shown in the red dashed line. There are some boulders on the beach. According to a fisherman this beach is not facing a coral reef.

Figure 59: Coastline of Rocky Point

Figure 60: Rubbles on the beach indicating existence of groin in the past

(a) Facing east

(b) Facing west

Figure 61: Beach on the east side in Rocky Point

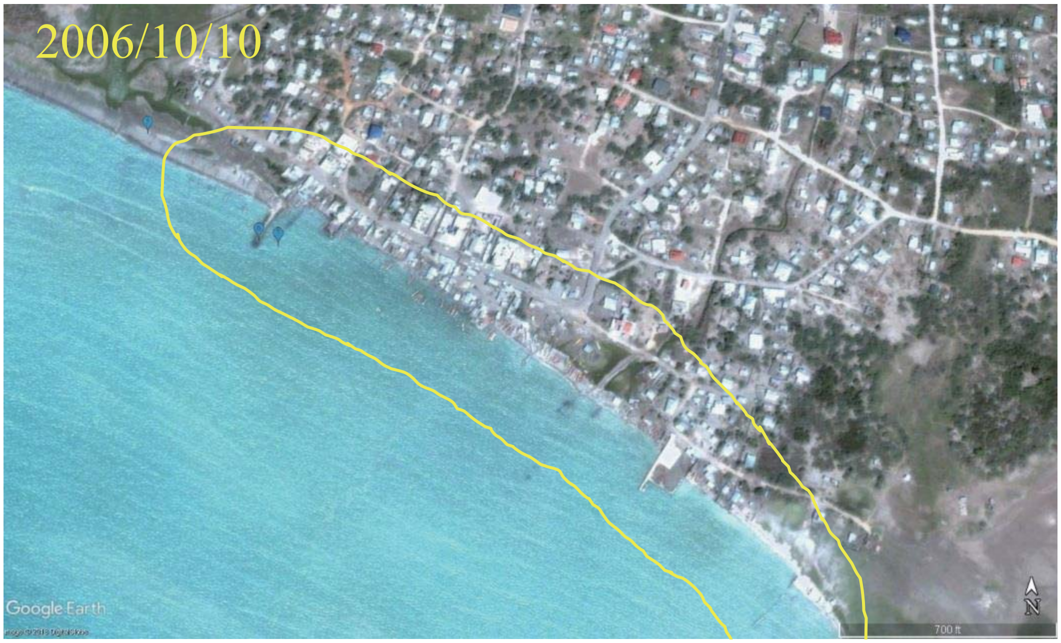
(a) Facing east

(b) Facing west

Figure 62: Beach on the west side in Rocky Point

Figures 61 and 62 show the east and west sides of beaches, respectively. The beach on the east side is covered with white sand while that on the west side with grey sand. The colour of the sediment is obviously different each other. These two facts that the shoreline is not straight and the sediment changes in the beach are not natural from the point of view of the coastal engineering. That made the author curious about this beach.

Figures 63 shows the comparison of the east beach in October, 2006 with that in January, 2017. There were some groins along the beach as shown in **Fig.** 63(a). The existing east beach covered with white sand was not visible on the photo in 2006. However, the wide beach was confirmed in the satellite photo dated on 17th of February, 2017. This beach extends from the



(a) 10th of October, 2006

(b) 4th of January, 2017

Figure 63: Comaprison of the east side beach

southeast coast up to the groin where two blue balloons are seen. Therefore, longshore sediment was transported from the southeast and trapped by the groin there. It is difficult to refer to whether or not the longshore sediment was transported beyond the groin to the northwest side.

(a) 10th of Octorber, 2006

(b) 6th of September, 2012

Figure 64: Comaprison of the southeast coast of Rocky Point

Figure 64 shows a wide range of the coastline from Rocky Point to the southeast beach. The coastal zone also drastically changed from 2006 to 2012. The water zone along the coastline within a marked area with the yellow curve in October, 2006 was filled with sand by September, 2012. Some event caused sediment transport from the southeast to the northwest during this period.

Figure 65: Coastline from the Rio Minho River to Rocky Point

The fact that the sediment on the west side differs from that on the east side indicates that there is another resource of the sediment. **Figure** 65 shows the wide range of coastal zone near Rocky Point in December, 2005. There is the Rio Minho River to the west in Clarendon Parish. The Rio Minho River is the longest river in Jamaica which extends 92.5 km in length. The river mouth was open and some sand bars were confirmed in the area in front of the river mouth.

Figure 66 shows the comparison of the coastal line between in 2003 and in 2005. The beach in the area indicated by the oval was extended offshore. However, the beach on the east side of the groin was not changed significantly. It means that sediment was transported from the west and trapped by the groin on the west side.

Between May, 2003 and December, 2005, some hurricanes passed over Jamaica. The biggest hurricane among them was Hurricane Ivan[10]. It hit Jamaica from 10th to 12th of September, 2004. Although it is not confirmed by the author that this event caused flooding and discharge of sediment into the water from the Rio Minho River mouth to Rocky Point, it is likely that a hurricane caused this kind of event. What caused sediment transport to the west coast of Rocky Point is not explained yet. However, it is reasonable to conclude that the grey sand might be replenished from the Rio Minho River.

(a) 26th of March, 2003

(b) 31st of December, 2005

Figure 66: Comparison of the west beach

7 Beaches in Jamaica

This section introduces a photo album of beaches in Jamaica. The photo album is available in the form of kmz files on-line at the NWA website. The URL is http://www.nwa.gov.jm/ and search the key word of 'album of coastline'. There are 13 kmz files of beaches in parishes and two of rivers. Locations are shown in **Fig.** 67. That includes about 180 locations. After opening kmz file(s), you click a balloon. Then you will see photos of beach or river. **Figure** 68 is an example showing photos of beach at the Morant Point in St. Thomas Parish. Most of the locations have a set of four photos. Two of them show the shoreline on both east and west sides at one beach or on both south and north. The other two photos show waves and sediment there.

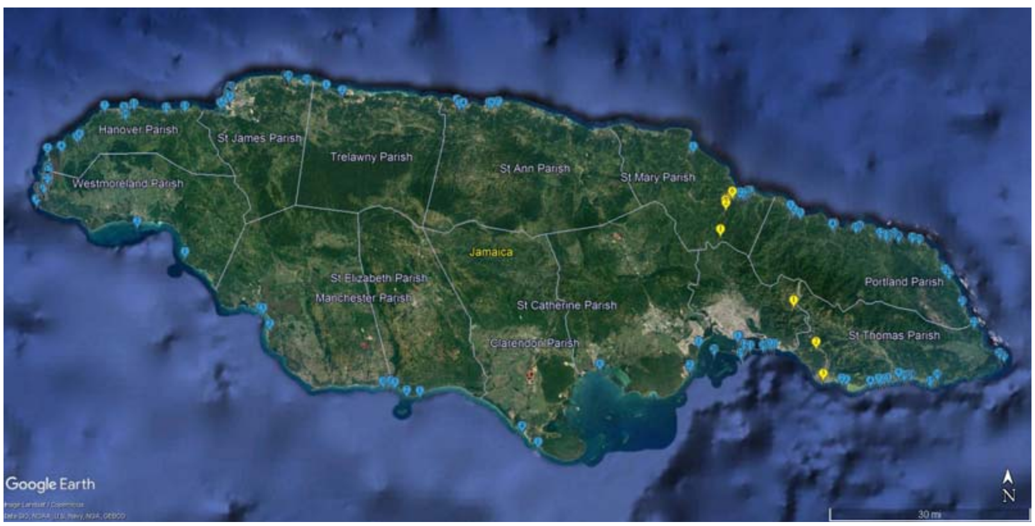


Figure 67: Locations of beaches where photos were taken.

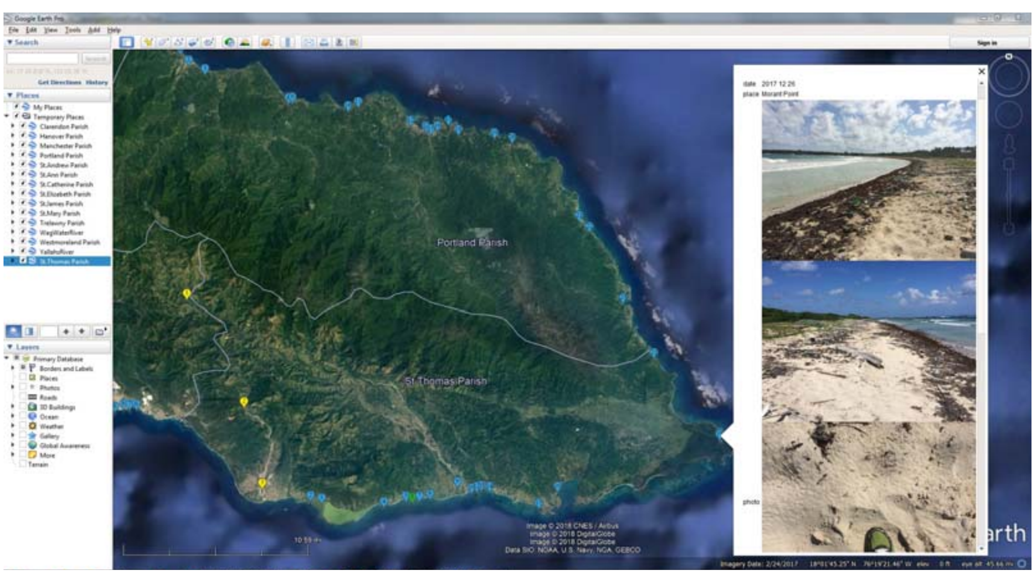


Figure 68: An example of photos of beach on Google Earth Pro

(a) facing east

(b) facing west

(c) waves on the beach

(d) sediment at beach

Figure 69: Beach in Alligator Pond on 18th of October, 2017


(a) facing east


(b) facing west


(c) waves on the beach


(d) sediment at beach

Figure 70: Beach in Alligator Pond on 10th of June, 2018

(a) facing east

(b) facing west

(c) waves on the beach

(d) sediment at beach

Figure 71: Beach to the east of Alligator Pond in Green bay

(a) facing east

(b) facing west

(c) waves on the beach

(d) sediment at beach

Figure 72: Beach to the east of Alligator Pond in Gautier bay

(a) facing east

(b) facing west

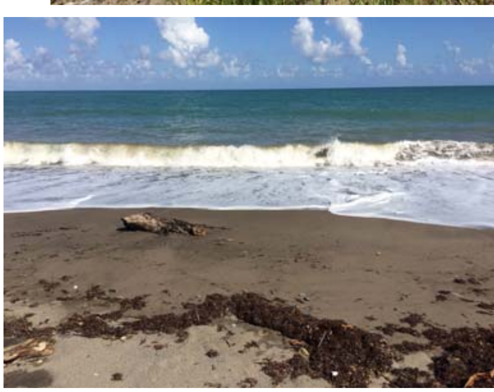
(c) waves on the beach

(d) sediment at beach

Figure 73: Beach at Alligator Pond River mouth

(a) facing east

(b) facing west



(c) waves on the beach

(d) sediment at beach

Figure 74: Beach in Annotto Bay

(a) facing east

(b) facing west

(c) waves on the beach

(d) sediment at beach

Figure 75: Beach in Annotto Bay town centre

(a) facing east

(b) facing west

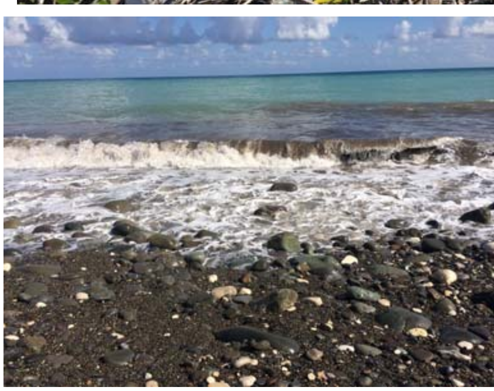
(c) waves on the beach

(d) sediment at beach

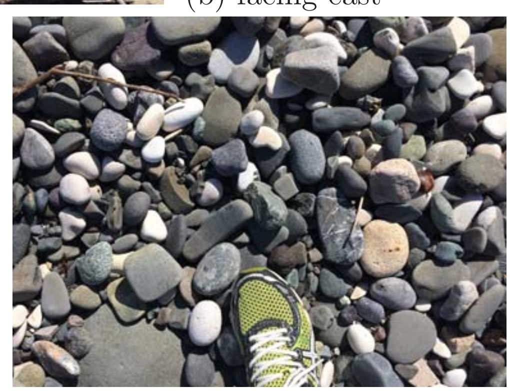
Figure 76: Beach in the south of Belmont, Westmoreland

(a) facing west

(b) facing east



(c) waves on the beach



(d) sediment at beach

Figure 77: Beach in the northeast of Buff Bay

(a) facing wesat

(b) facing east

(c) waves on the beach

(d) sediment at beach

Figure 78: Beach in the east of Bull's Bay, Hanover

(a) facing east

(b) facing west

(c) waves on the beach

(d) sediment at beach

Figure 79: Beach in Hellshire

(a) facing west

(b) facing east

(c) waves on the beach

(d) sediment at beach

Figure 80: Beach near Hope Bay

(a) facing west

(b) facing east

(c) waves on the beach

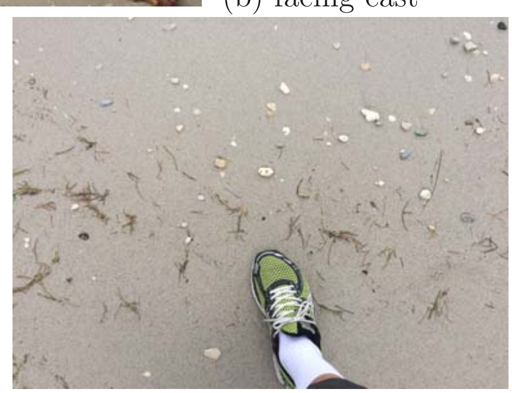
(d) sediment at beach

Figure 81: Beach near Swift River in Hope Bay

(a) facing west

(b) facing east

(c) waves on the beach



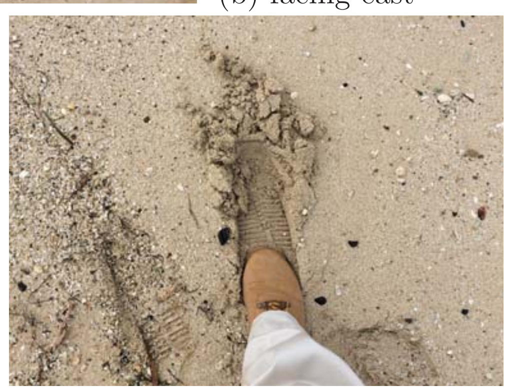
(d) sediment at beach

Figure 82: Beach in Hopewell

(a) facing west

(b) facing east

(c) waves on the beach



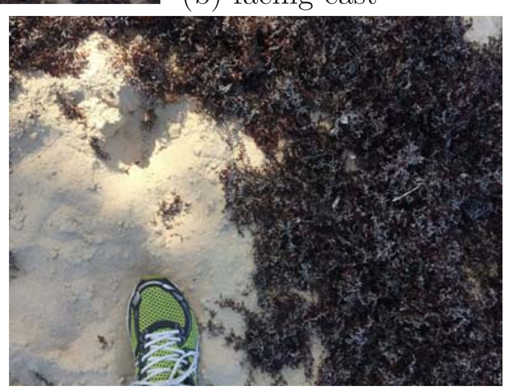
(d) sediment at beach

Figure 83: Beach in Jackson Bay

(a) facing west

(b) facing east

(c) waves on the beach



(d) sediment at beach

Figure 84: Beach in Long Bay, Portland Parish

(a) facing west

(b) facing east

(c) waves on the beach

(d) sediment at beach

Figure 85: Dead End Beach in Montego Bay

(a) facing west

(b) facing east

(c) waves on the beach

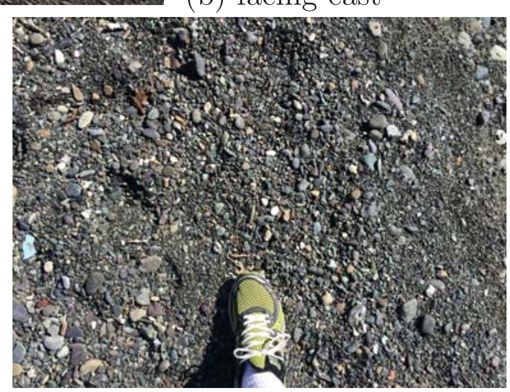
(d) sediment at beach

Figure 86: Dump Beach in Montego Bay

(a) facing west

(b) facing east

(c) waves on the beach

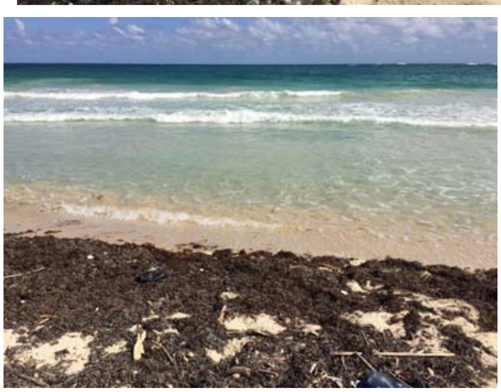


(d) sediment at beach

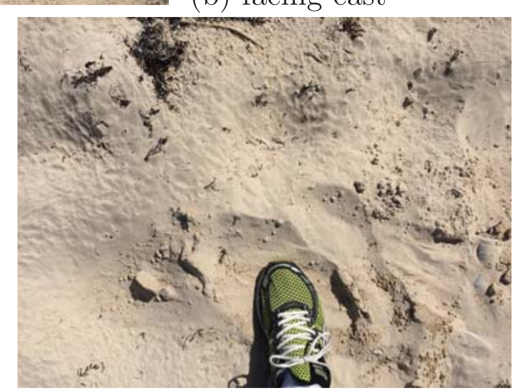
Figure 87: Beach in Morant Bay

(a) facing west

(b) facing east



(c) waves on the beach

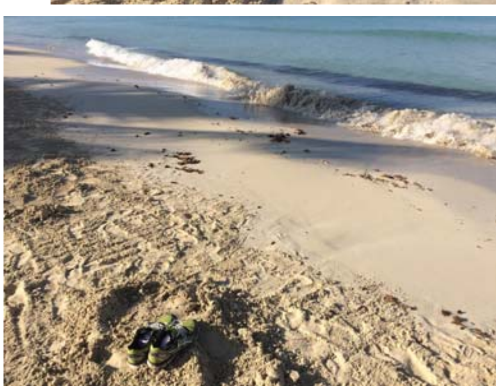


(d) sediment at beach

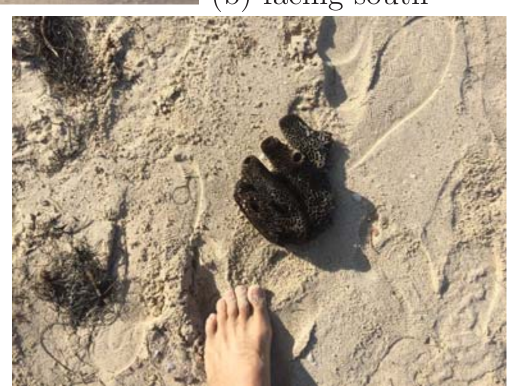
Figure 88: Beach in Morant Point

(a) facing north

(b) facing south



(c) waves on the beach



(d) sediment at beach

Figure 89: Beach of Seven Mile Beach in Negril

(a) facing east

(b) facing west

(c) waves on the beach

(d) sediment at beach

Figure 90: Beach in Old Harbour


(a) facing north


(b) facing south

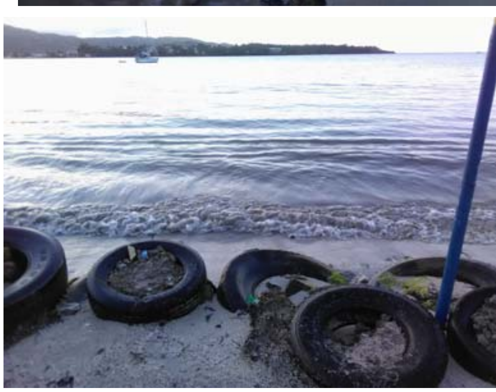

(c) waves on the beach


(d) sediment at beach

Figure 91: Beach in Orange Bay


(a) facing west


(b) facing east


(c) waves on the beach

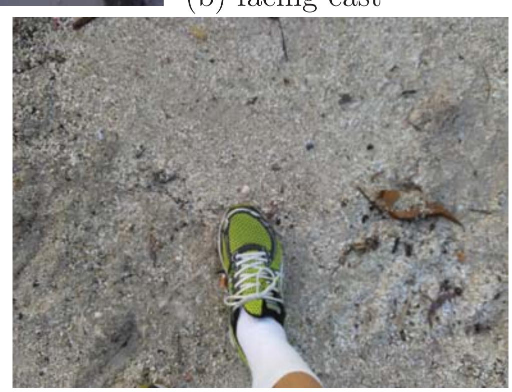

(d) sediment at beach

Figure 92: Beach in Port Antonio

(a) facing east

(b) facing west

(c) waves on the beach

(d) sediment at beach

Figure 93: Beach in Port Royal.


(a) facing east


(b) facing west


(c) waves on the beach


(d) sediment at beach

Figure 94: Beach on the east side of Rocky Point

(a) facing east

(b) facing west

(c) waves on the beach

(d) sediment at beach

Figure 95: Beach on the west side of Rocky Point


(a) facing west


(b) facing east


(c) waves on the beach


(d) sediment at beach

Figure 96: Beach in Runaway Bay

(a) facing west

(b) facing east

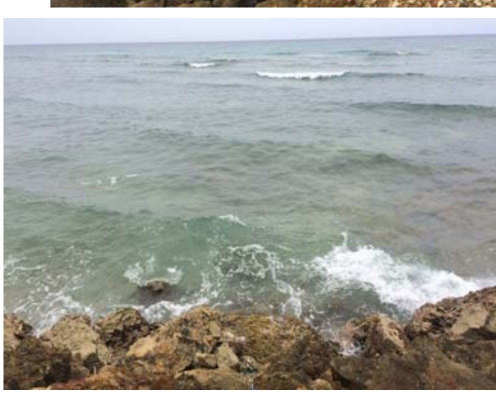
(c) waves on the beach

(d) sediment at beach

Figure 97: Beach in Runaway Bay

(a) facing west

(b) facing east



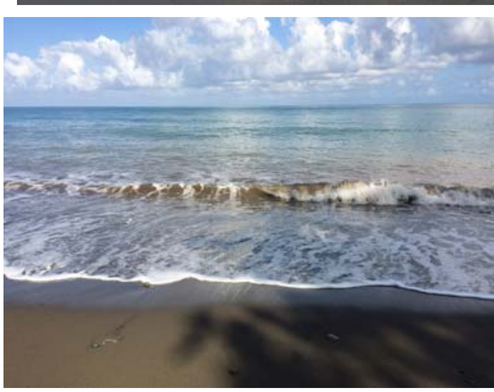
(c) waves on the beach

(d) sediment at beach

Figure 98: Coastline in Sandy Bay

(a) facing west

(b) facing east



(c) waves on the beach

(d) sediment at beach

Figure 99: Beach in St. Magarets Bay


(a) facing west


(b) facing east


(c) waves on the beach

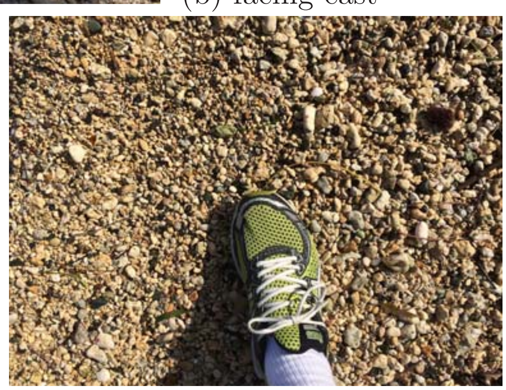

(d) sediment at beach

Figure 100: Beach in Williams Field, Portland Parish

8 Concluding Remarks

Coastal engineering is an indispensable subject for island countries. Jamaica needs more coastal engineers for both sustainable development and preserving the environment. Engineers need to learn equations in the coastal engineering for practical use. The author wishes this concise textbook helps Jamaican engineers and students in civil engineering have interest in the water waves and beach erosion

In terms of beach erosion, beaches can be in the state of static equilibrium or dynamic equilibrium. In order to keep beaches the state of dynamic equilibrium or to recover beaches from coastal erosion, we should focus on resources of sediment. Beaches need sand replenishment. In the case of open sea beaches, river training should be conducted to open the river mouth. As for coral reef beaches, maintaining and restoring the healthy coral reefs is essential.

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